## Unit 8 Homework Problems

## Learning Goals:

F. 8 Apply the impulse-momentum theorem to a real-life situation, quantitatively or qualitatively, even if the force is not constant.

8-1) A Pickup truck attempts to tow a car. The car and the truck are connected by a 2 m long rope.
(a) At first, the truck doesn't pull hard enough, and the car doesn't move. Compare the force exerted on the rope by the truck's bumper, $F_{R-T}$ to that exerted on the truck's bumper by the rope $F_{T-R}$. Also compare the force exerted on the car's bumper by the rope $F_{C-R}$ to that exerted on the rope by the car's bumper $F_{R-C}$. Compare all four forces to each other and list them in ascending or descending order.
(b) The truck finally pulls hard enough so that the car begins to move. Assume that the car is being accelerated with a constant acceleration of $0.5 \mathrm{~m} / \mathrm{s}^{2}$ and that the 2 m long rope we mentioned above is one of those fabulously unbelievable massless ropes that exist in reality only in the heads of theoretical physicists. As in part (a) compare all four forces to each other and list them in ascending or descending order.
(c) Now let's be a little more realistic and replace that massless 2 m long rope by a realistic rope, one that could be found under the front seat of the pickup truck. Let's assume that the rope has a mass of 1.0 kg . As in part (c), assume that the car is being accelerated with a constant acceleration of $0.5 \mathrm{~m} / \mathrm{s}^{2}$. As before, compare all four forces to each other and list them in ascending or descending order.

8-2) IMPULSE AND MOMENTUM CHANGE: The diagram to the right shows the momentum vectors for a particle before and after it is hit with a sharp blow. The particle's mass is 2.00 kg and its initial speed is $0.707 \mathrm{~m} / \mathrm{s}$. It has an initial momentum vector $\vec{p}_{1}$ that points in a direction that is $45.0^{\circ}$ from the $x$-axis as shown. After being given a sharp blow it has a final momentum, $\vec{p}_{2}$, that points along the positive $y$-axis with a magnitude of $1.00 \mathrm{~kg}-\mathrm{m} / \mathrm{s}$.

(a) Use the two arrows to construct a vector representing the momentum change, $\Delta \vec{p}$, that resulted from the blow.
(b) Write out the initial and final momentum as vector components using unit vector notation $(\hat{x}$ and $\hat{y})$. Then use vector subtraction to show that the momentum change, $\Delta \vec{p}$, is given by

$$
\Delta \vec{p}=\left(-1.00 \mathrm{~kg} \frac{\mathrm{~m}}{\mathrm{~s}}\right) \hat{x}
$$

(c) What was the direction of the blow?
(d) If the blow lasted for 0.100 s , what was the average force on the object?

8-3) You have been hired to evaluate the safety of a new 3-meter diving platform that is under development. One concern is that large forces on the ankles and knees might injure the diver if the board is too stiff to give a long spring time. You digitized a series of frames at a full 30 fps and called the digital movie file $d$ son019.mov. I've also created a $d$ son019. cmbl file for you to use, with the proper scaling and origin. You proceed to estimate the maximum height above the diving board of his jump onto the board for his final push off. You then use that information to calculate his velocity when his feet first touch the board as part of the final push off.

Hint: While you could collect data for frames 26 through 61, you can save yourself quite a bit of time by recognizing that you really only need information for three key frames - 35, 46, 54 - and only time data; you don't need position data.
(a) Draw three free body diagrams showing all the forces on the diver as he is:

1) rising (between frames 26 and 35),
2) turning around at the top of his jump (between frames 34 and 36), and
3) falling back toward the board (between frames 35 and 46).
(b) Assuming the lines shown in the title screen to be used for scaling are accurate, determine the diver's velocity just as his feet touch the board in frame 46. Show any equations you used as well as your calculations.

In the next phase of your study, you turn to an examination of the times and forces involved in the diver's encounter with the end of the board in frames 47 through 61 (times 1.566 s and 2.033 s ). For a given initial velocity the difference between the forces associated with a long spring time and a short one can be readily experienced. You decide to just leap straight up in the air and then land gently by bending your knees. You then repeat the exercise with your knees stiff (locked). (Yes, I would like you to actually, physically do this leaping exercise.)
(c) Describe which situation causes the floor to exert the most force on your joints (ankles and knees). Discuss what theory would help explain your observations.

To find the average forces on the test jumper's ankles you examine movie frames 47 through 61 (times 1.566 s and 2.033 s ).
(d) Draw three free body diagrams showing all the forces on the diver as he is:

1) sinking into the board,
2) turning around, and
3) being thrown back up by the board.
(e) How long did it take the diver to slow down from his initial velocity (when he first touches the end of the 3-meter board) until his downward velocity is zero (at his lowest point on the board)? Explain how you estimated this time.
(f) If the mass of the diver is 64 kg what is his momentum change?
(g) What is the average force on his ankles during the period of time when the diving board is slowing him down to zero velocity?
(h) How do you know this average force? Did you measure it directly, or did you use an understanding of a physical law to deduce it? If you used a physical law, explain which one it was.

8-4) In the summer of 1969 NASA's Apollo 11 mission to send the first men to the moon took place. This was a very complex mission in which a Saturn V rocket containing a command module (CM) and lunar module (LM) was launched from the Earth and eventually placed into orbit around the Moon. As the command module with astronaut Michael Collins aboard orbited the moon, astronauts Neil Armstrong and Buzz Aldrin transferred to the LM for the final descent to the Moon's surface. After conducting a series of experiments on the Moon they re-entered the LM, which then ascended for re-docking with the orbiting Command Module and eventual return to Earth. One extremely important phase of this project and the subsequent Apollo missions (Apollo 12-17) was to set up the LM for ascent so astronauts could return to Earth.

The President wants to re-institute manned missions to the moon. You have been hired to help with planning the design of a new Lunar Module capable of delivering the thrust forces needed to lift it into lunar orbit for docking with a Command Module. You want to begin the planning by learning as much as possible about the LM ascents that took place during the manned Apollo missions. Unfortunately, a major flood in the Washington D.C. area during 2019 destroyed some of NASA's Apollo mission archives.

Suppose you have obtained three items of public information pertaining to a lunar ascent:
First, you have a set of video frames showing the ascent in the Apollo 11 mission which have been digitized at a rate of 10 fps using a QuickTime format that can be opened by the Logger Pro software. It has the file name nasa003.mov. (Fortunately, NASA arranged to leave a remotely controlled camera behind that was capable of radioing movie frame data back to the command module!). Several frames of the movie are shown below.

Second, you have a data file for the vertical height, $y$, in meters vs. time, $t$, in seconds for the LM ascent. Information for the LM height as a function of time was hard to extract from the digital movie of the lunar ascent. This is because the camera is zooming back so the apparent size of the LM as well as any fixed points on the LM launch pod or the moon are changing location from frame-to-frame. However, the wise engineer used a "moving origin" fixed on one of the pod legs and performed frame-by-frame scaling to obtain intelligible data for $y$ vs. $t$. Assume for the sake of the exercise that follows that her data file was created by one of the original Apollo engineers. The data file name is nasa003.xlsx.

Third, you have information about the LM found in a NASA Apollo 11 Lunar Landing Mission Press Kit distributed by Aerospace Resources International in Silver Spring, MD before the flood.
(a) Please draw a free body diagram showing all the forces on the LM as it ascends and explain what equations you could use to determine the thrust force as a function of the net force and the gravitational force on the LM.
(b) Use the $y$ vs. $t$ data in the nasa003.xlsx file to show that the LM lift off acceleration is constant and to find the magnitude and direction of the acceleration.
(c) Based on your results, is the thrust force constant? What is the evidence for your answer?
(d) What is the liftoff mass of the LM with full fuel carrying two 180 lb astronauts in space suits?
(e) Find the net force, the gravitational force, and the average thrust force on the LM. Hint: Don't forget that the gravitational constant is different on the moon. See the DATA shown below for details.

What impulse does the LM experience during the first 1.9 seconds of its ascent?


## DATA FROM THE NASA003.XLS FILE

| Apollo 11 LM Height vs. time |  |  |
| :---: | :---: | :---: |
| Frame | $\boldsymbol{t}(\mathbf{s})$ | $\boldsymbol{y}(\mathbf{m})$ |
| 1 | 0.000 | 3.98 |
| 2 | 0.100 | 4.31 |
| 3 | 0.200 | 4.66 |
| 4 | 0.300 | 4.99 |
| 5 | 0.400 | 5.32 |
| 6 | 0.500 | 5.78 |
| 7 | 0.600 | 6.72 |
| 8 | 0.700 | 6.96 |
| 9 | 0.800 | 7.38 |
| 10 | 0.900 | 8.15 |
| 11 | 1.000 | 8.67 |
| 12 | 1.100 | 9.07 |
| 13 | 1.200 | 9.94 |
| 14 | 1.300 | 10.73 |
| 15 | 1.400 | 11.42 |
| 16 | 1.500 | 11.97 |
| 17 | 1.600 | 13.10 |
| 18 | 1.700 | 13.37 |
| 19 | 1.800 | 14.04 |
| 20 | 1.900 | 15.22 |

## APOLLO PRESS KIT DATA ON THE LM

Height: 22 feet 11 inches
Width (diagonally across the landing gear): 31 feet Ascent Stage Weight (w/o propellant or crew): 4,804 lb* Ascent Propellant Weight: 5,214 lb*
Extra vehicular Space Suit Weight (worn by the commander and lunar module pilot): $35.6 \mathrm{lb}^{*}$
Gravitational Strength on the Moon $(g): 1.62 \mathrm{~N} / \mathrm{kg}$
*(Note: You should infer that since this is a press kit, weights in lb are Earth weights so that at the surface of the Earth, on object with a mass of 1 kg has a weight of 2.20 lb )

8-5) Far in space, where the gravitational force is negligible, a 425 kg rocket traveling at $75.0 \mathrm{~m} / \mathrm{s}$ in the positive $x$-direction fires its engines. The figure below shows the thrust force as a function of time. The mass lost by the rocket during these 30.0 s is negligible.
(a) What impulse does the engine impart to the rocket?
(b) At what time does the rocket reach its maximum speed?
(c) What is the final velocity of the rocket?


8-6) Problem 8.10.1 from the Activity Guide.

