Part III: Two Waves Traveling Together, Out of Phase

Imagine two waves traveling in the same direction on a stretched string. They have the same velocity and frequency, but are *out of phase*. This means the equations for each wave are given by:

$$y_1(x,t) = Y\sin(kx - \omega t) \text{ and } y_2(x,t) = Y\sin(kx - \omega t - \phi_0)$$

1. Use the principle of superposition to find the resultant wave. Also use the trigonometric identity $\sin(\alpha) + \sin(\beta) = 2\sin\left[\frac{1}{2}(\alpha+\beta)\right]\cos\left[\frac{1}{2}(\alpha-\beta)\right]$.

2. Describe the result. What would you actually see on the string? Could you tell it was the sum of two traveling waves just by looking at it?

3. Open the Mathematica file "***phys301-superposition.nb***". (Reminder: You can access Mathematica using Westminster Anywhere, or simply log in to our classroom computers.) To "Run" a cell in Mathematica, with your curser in the cell, type Shift+Enter. This tells Mathematica to "do" the commands in the cell, such as define a variable as a certain number. For each of the three cells, in order, place your cursor in the cell and type Shift+Enter. An animation will display the waveform for each individual wave (in colored, dashed lines) and the waveform for the resultant wave (in black, solid line – what you'd actually see on the string).

4. For each possible value of ϕ_0 below, **predict** the amplitude of the resultant wave in terms of *Y*, the amplitude of each individual wave.

Phase Difference ϕ_0 (radians)	Predicted Amplitude of Resultant Wave
0.0	$2Y$
2.0	
π	
$4\pi/3$	
2π	
$31\pi/32$	

5. Test out your predictions by changing the value of Φ_0 in the first cell and typing Shift+Enter each time. Make a note to the side of the table if any predictions were incorrect.

When the amplitude of the resultant wave is twice that of each individual wave ($2Y$), we say the two waves are "fully constructive." When the amplitude of the resultant wave is zero, we say they are "fully destructive." For all other phase shifts, the result is somewhere in-between, or "intermediate" interference.

Part IV: Two Waves Traveling In Opposite Directions
Now imagine two waves traveling on the same stretched string. This time, the only difference between the two waves is the *direction* of their travel. This means the equations for each wave are given by:

$$y_1\left(x,t\right) = Y\sin\left(kx - \omega t\right) \text{ and } y_2\left(x,t\right) = Y\sin\left(kx + \omega t\right)$$

6. Which wave is traveling to the left? Which wave is traveling to the right?

7. Use the principle of superposition to find the resultant wave. See trig identity on previous page!

8. Describe the result. Does this describe a traveling wave, or something else? What do you predict this resultant wave might look like?

9. Let's modify our Mathematica notebook to display this case. Change ϕ_0 back to zero in the first cell. Change the minus sign in the equation for $y_2\left(x,t\right)$ to a plus sign. Be sure to type Shift+Enter when done with each cell. Observe how the animation changes. You may want to slow it down or pause it and use the slider to step through slowly.

You have created a *standing wave*. There should be some places along the *x*-axis where the string *never moves*. These are called *nodes*. Halfway in between the nodes are places along the *x*-axis where the amplitude of the resultant wave is a maximum. These are called *antinodes.*

10. Using the general equation for the resultant wave that you derived in this activity, determine the location of the nodes. Hint: This is the physical location in the *x*-direction where the amplitude is always zero. For what arguments will the sine of an angle equal zero? (There are many possible answers.)

11. For the specific wave you created in Mathematica, where do you expect the nodes to be? Is that where they are?

In practice, we can create standing waves by using *boundary conditions*.