Beats

So far, our study of interference has focused on some very special cases:

- Two waves traveling together, identical except for being out of phase. The result was another traveling wave.
- Two waves traveling in opposite directions, identical except for the direction of their travel. The result was a standing wave.

What happens when we allow the frequencies of the two interfering waves to differ from one another? Like with the previous two examples, Mathematica is a powerful tool to investigate this question.

1. Open the Mathematica file "***phys*301-*beats.nb***".
2. The first cell defines the values of the variables we'll use, and the second cell defines the equations for the two waves that will interfere with one another. Notice that I've set these waves to travel at the same speed, but with the option to have different frequencies. Type Shift+Enter in each of these cells to run them.
3. The third cell shows an animation of the two individual waveforms (y vs. x, dashed lines) and their sum (black, solid line) with time. With these default values, the two frequencies of the waves are the same. What do you observe?

4. Change ω_2 to be close, but not quite, the same as ω_1. For now, try setting ω_2 to be 90% the value of ω_1. Be sure to type Shift+Enter in the cell when done. The animation will automatically update. What happens?

5. We should investigate how the wave's amplitude changes with time. The fourth cell produces a (static) plot of the y-position at x = 0 m as a function of time. It does not update automatically; click in the cell and type Shift+Enter to update the output. Roughly sketch the plot below and describe what you see.

The interesting pattern you found appears to be two separate sinusoidal patterns superimposed on top of one another. There is the frequent up-and-down pattern that seems similar to that of any of the one individual waves alone. The overall amplitude, however, is increasing and decreasing in a sinusoidal fashion, but over a much longer period of time. This is one form of *modulation*, namely amplitude modulation from which AM radio gets its name.

If these were sound waves, rather than the vertical axis representing y-position (like for a wave on a string), it would represent ΔP, a change in pressure from equilibrium. A positive value of ΔP corresponds to compression of a packet of air, and a negative value of ΔP corresponds to an expansion. The large change in amplitude corresponds to a noticeable variation in the intensity (volume) of the sound. The slow increases and decreases in the intensity are called *beats*.

6. At this point, your professor will demonstrate this phenomenon with real sounds. Record your observations here.

7. For the waves you've represented in Mathematica, devise a method to measure the frequency of this *longer period* wave (the changes in intensity). This is the frequency from, for example, one peak in amplitude to another. To measure coordinates from a plot in Mathematica, you can select the "**Drawing Tools**" command under the "**Graphics**" menu and select the "**Get Coordinates**" button (which looks like dashed crosshairs) from the popup window. You can click and drag the corners of the graph if you would like to make it bigger (or change "`ImageSize → 450`" in the cell to a larger value).

8. How does this frequency compare to the individual frequencies of the two waves? Is it the average between the two? The sum? The difference? The product? The ratio? What else could I possibly make up?

9. Summarize your findings by writing an equation which relates f^{beat}, f_1, and f_2, where f_1 and f_2 are the respective frequencies of each individual wave.

10. Given your answer to the previous questions, predict what the y vs. t graph would look like if you brought the frequencies closer together (e.g., $\omega_2 = 0.95\ \omega_1$)? Sketch your (rough) prediction below.

11. Try it out. How good was your prediction? What would you hear if these were sound waves?

12. Predict what the y vs. t graph would look like if you brought the frequencies further apart (e.g., $\omega_2 = 0.80\ \omega_1$)? Sketch your (rough) prediction below.

13. Try it out. How good was your prediction? What would you hear if these were sound waves?

In a sense, music is just the superposition of many, many sound waves. Playing one note on a violin produces a combination of the fundamental mode and higher harmonics simultaneously. The combinations of these higher harmonics depend on the instrument itself, and that is the difference between a poor quality vs. professional instrument. Chords are produced by playing different frequencies of notes simultaneously. Some interfere in a pleasant way, and some sound just gross. This is the very mathematical side of music theory.