**Part I: Blackbody Radiation**

Any object with a temperature above absolute zero will radiate energy in the electromagnetic spectrum. It creates a "continuum spectrum," because there is continuous emission at all wavelengths. A perfect emitter/absorber is called a "blackbody," because it can be idealized as a cavity covered with perfectly black (perfectly absorbing) walls. The continuum spectrum of stars are approximately blackbody curves. These curves were well measured, and described mathematically, before the quantum mechanical reasons for them was figured out. Let's investigate how these curves work before we talk about the quantum mechanics behind them.

1. We'll use the PhET Interactive Simulation called Blackbody Spectrum. To open the simulation, go to https://phet.colorado.edu/sims/html/blackbody-spectrum/latest/blackbody-spectrum_en.html. You can create different temperature stars by simply moving the "Temperature" slider up or down.

2. The Sun has a surface temperature of about 5800 K, the default value for this simulation. Notice that there are approximately equal amounts of power radiated in the red (long wavelength) and blue (short wavelength) portions of the spectrum. Slightly more power is emitted in the green and yellow region. This causes the sun to appear mostly white, maybe a little yellow/orange.

3. Increase and decrease the temperature to simulate emission from a different temperature star. What ***two*** things change about the blackbody curve when temperature is *increased*? (You may need to click on the or icon for the vertical and horizontal axes.)

4. Wien's law, determined experimentally, states that $\lambda_{\text{max}} T = 2.898 \times 10^{-3}\,\text{m} \cdot \text{K}$. What temperature should you choose to create a spectrum whose peak wavelength is 446 nm?

5. Try out your prediction. Did it work? (Be sure to click on the "Graph Values" checkbox.)

6. If a star emits significantly more blue light than red, it will appear blueish. If it emits significantly more red light than blue, it will appear reddish. Is a reddish star hotter or colder than a blueish star? Be sure to cite your supporting evidence from the simulation.

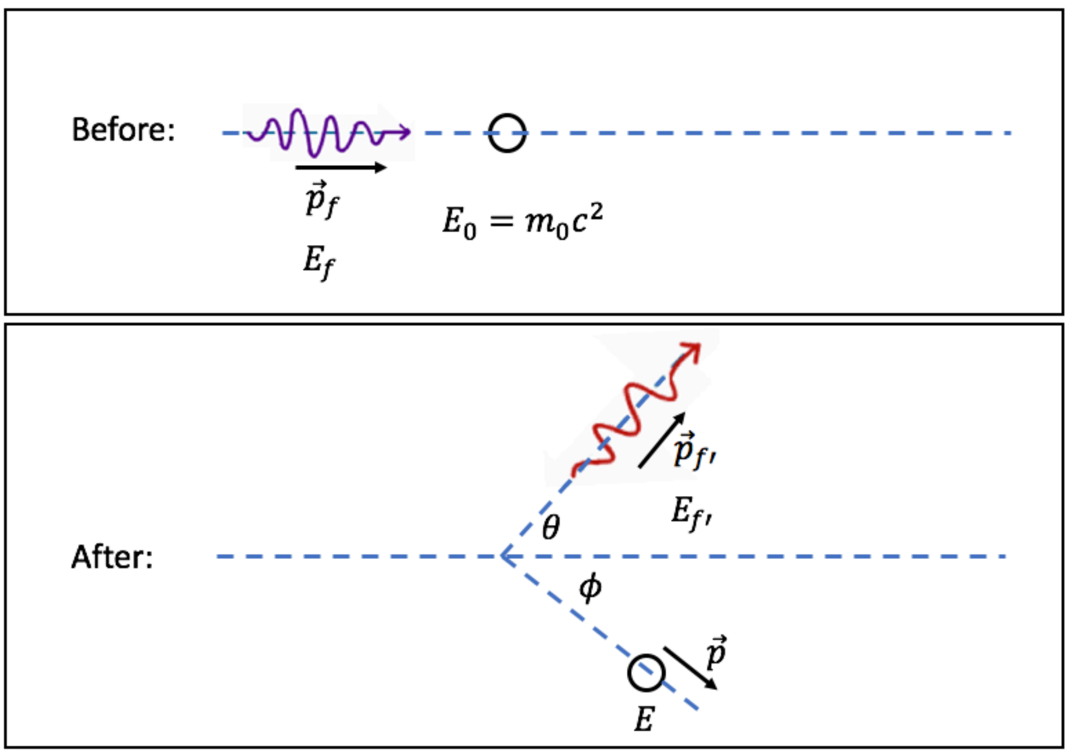
Note: As you change the temperature, the image of the star at the top of the simulation appears to grow larger or smaller. All stars (except the Sun) cannot be "resolved" by our telescopes. They are little pinpoints of light, and that light gets "smeared out" by the atmosphere or by the properties of the optics being used. A seemingly "bigger" star in a telescope image is not necessarily bigger in size (radius), but it is *brighter*. (In this case though, Sirius A is actually bigger, but you *cannot* tell that just by an image.)

**Part II: Derivation of the Compton Shift**

If light were a wave, scattered light would keep its same wavelength. But for very small wavelengths, like x-rays, being scattered through some materials, like graphite, this is not true. The wavelength changes when scattered. Moreover, the change in the wavelength, $\Delta\lambda$, ,depends on the angle of scattering. In graphite, the valence electrons are very loosely bound to their atoms. They are basically free electrons that an incoming photon (particle of light) can collide with. The photon transfers energy and momentum to the electron. This lost energy means the photon emerges from the collision with a longer wavelength. We will find the expression for $\Delta\lambda$ as a function of scattering angle by treating this process as the elastic relativistic collision of two particles, the photon and the electron. We'll use conservation of energy and conservation of relativistic momentum.

**The Setup**

Imagine the incoming photon (traveling from left to right) has initial frequency $f$, energy $E_f$, and initial momentum $\vec{p}_f$. The electron is at rest and therefore only has rest energy $E_0 = m_0c^2$. After the collision, the photon emerges at an angle $\theta$ with respect to the horizontal with new frequency $f'$, new energy $E_{f'}$, and new momentum $\vec{p}_{f'}$. Note the $f'$ (f-prime) in the subscripts. The electron emerges at angle $-\phi$ with respect to the horizontal with energy $E$. Nobody cares about the electron (sorry little dude), so we'll try to eliminate those variables.



1. Write an equation for Conservation of Momentum using the variables in the figure. All three terms should be vectors.

2. Write an equation for Conservation of Energy using the variables in the figure above. All four terms are scalars.

3. Using your Conservation of Energy equation, isolate the energy of the electron after the collision, $E$, on one side. Square both sides.

4. From here on out, keep the term $\left(E_f - E_{f'}\right)$ together as one term, because what's what we're interested in – the difference in the photon's energy before and after. Expand the square so that you now have three terms on one side of the equation, and $E^2$ on the other side:
$$\left(E_f - E_{f'}\right)^2 + 2E_0\left(E_f - E_{f'}\right) + E_0^2 = E^2$$

5. Note that $E_0 = m_0c^2$ if you haven't already made that substitution. Subtract $\left(m_0c^2\right)^2$ from both sides.

6. What is the relativistic energy-momentum relation?

7. Use the relativistic energy-momentum relation to write your $E^2 - \left(m_0c^2\right)^2$ side of the equation in terms of the momentum of the electron.

8. We still have a lot of factors of "energy of photons" in our equation, but if we're ever going to combine this with our Conservation of Momentum equation, we'll need to express all those in terms of momentum. First, divide both sides of your equation by $c^2$ such that all of your $E_f$ and $E_{f'}$ terms can be expressed as $E_f/c$ and $E_{f'}/c$ respectively.

9. Using the relativistic energy-momentum relation and the fact that photons have zero rest mass, what is the relationship between $E_f/c$ and $p_f$?

10. Substitute in your relationship so that your conservation of energy equation now only includes the terms $p_f$, $p_{f'}$, $m_0$, $c$, and $p$. Call this Equation A.

11. We are partway there, but as previously mentioned, we don't care about the electron and its momentum. We'll use our Conservation of Momentum equation to solve for $p^2$ and substitute that into Equation A. So, first, solve your Conservation of Momentum equation for $\vec{p}$.

12. Then, square both sides of your Conservation of Momentum equation. Reminder! The product of two vectors is the scalar dot product: $\vec{a} \cdot \vec{b} = ab\cos\theta$, where $\theta$ is the angle between the two vectors. You should end up with three scalar terms on one side and $p^2$ on the other side. Call this Equation B.

13. Combine Equations A and B to eliminate the momentum of the electron. Simplify until you get to an equation that has $\frac{1}{p_{f'}} - \frac{1}{p_f}$ on one side.

14. We're so close! Our final step is to relate $p_f$ and $p_{f'}$ to $\lambda$ and $\lambda'$, respectively. Use your previous expression relating $E_f$ and $p_f$ along with the relationship between energy and wavelength, $E_f = \frac{hc}{\lambda}$. Solve for $\lambda' - \lambda$ (which is equivalent to $\Delta\lambda$).

The term $\frac{h}{m_0 c}$ contains only fundamental constants. It is given the special name "the Compton wavelength of the electron" and abbreviated $\lambda_c$. The Compton wavelength is 2.43 pm (picometers). One picometer is $10^{-12}\,\mathrm{m}$. Your final equation can then read:

$$\Delta\lambda = \lambda_c \left(1 - \cos\theta\right).$$

15. A 240 pm X-ray is incident on a calcite target. Find the wavelength of the X-ray scattered at a 45 degree angle. What percentage of the original wavelength is the wavelength shift?

16. Where would you place your detector (at what angle) to detect zero shift in wavelength?

17. Where would you place your detector (at what angle) to detect the maximum $\Delta\lambda$?

18. Few particles (less than 5%) are scattered at this angle (this is called "backscattering") and will thus require very sensitive detectors. Detectors are also limited in their resolution; that is, how small of a change in $\lambda$ can be detected. Imagine you can detect a 0.01% change in $\lambda$ for backscattered radiation. What is the maximum wavelength for which you could detect this effect? In what range of the electromagnetic spectrum is this?

Note that in addition to you not being able to measure the wavelength shift, a photon with too low of energy may not be able to knock the valence electron out of the atom anyway (or might use all its energy doing so). In the low-energy limit (photon energy much less than rest mass energy of the particle, or in other words, wavelength of light much greater than Compton wavelength of particle), this situation is called Thomson Scattering, and there is effectively no change in the wavelength of the light. Essentially, the charged particle accelerates and emits radiation at the same frequency as the incident wave.