Part I: Various Expressions for the de Broglie Wavelength

The de Broglie wavelength of a particle is Planck's constant divided by momentum: $\lambda = \frac{h}{p}$.

For relativistic particles, $p = \gamma_p mv$. You can approximate the momentum using the classical formula, $p = mv$, and be good within 2% for particles moving at less than $v = 0.20c$.

Often, you'll see information about particles expressed in terms of the *total, kinetic, and/or rest-mass energies* of the particles. Recall the Energy-Momentum relation from special relativity: $E^2 = (pc)^2 + E_0^2$.

1. Solve the Energy-Momentum relation for momentum.

2. Rewrite the de Broglie wavelength in terms of total energy and rest mass energy instead of momentum.

3. If the total energy of the particle is much greater than the rest mass energy, how does your expression simplify? Is this approximation good for a particle moving very quickly, or very slowly? Does it look familiar?

4. Sometimes you might be given the kinetic energy of a particle, instead of its total energy. Rewrite your expression from Question 2 in terms of kinetic energy and rest mass energy. This will be a useful expression when both energies must be taken into account.

Part II: Calculating de Broglie Wavelengths

$$p = \gamma_p m u \qquad \gamma_p = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}} \qquad E_0 = mc^2 \qquad E = \gamma_p mc^2 \qquad K = \left(\gamma_p - 1\right)mc^2$$

$$h = 4.14\times10^{-15}\,\text{eV}\cdot\text{s} \qquad c = 3.00\times10^8\,\frac{\text{m}}{\text{s}} \qquad hc = 1240\ \text{eV}\cdot\text{nm}$$

Rest energies of various particles:

- Electron: 0.511 MeV
- Proton: 938 MeV

1. Using your equation from Part I, Question 2, find the de Broglie wavelength of a proton whose kinetic energy is 2.0 MeV? Verify your result using the equation from Part I, Question 4. (Note: a femtometer (or "fermi") is 10^{-15} m, which is a convenient unit for particle physics.)

2. What is the de Broglie wavelength of an electron that is moving at 3% of the speed of light, $v = 0.03c$?

3. At what speed does a proton have a 6.0 fm wavelength (about the size of an atomic nucleus)? Give your answer in units of c.

Proton KE = 45 MeV

$$\lambda = \frac{hc}{\sqrt{E^2 - E_0^2}} = \frac{(1240\ \text{eV}\cdot\text{nm})(1\ \text{MeV}/10^6\ \text{eV})}{\sqrt{(983\ \text{MeV})^2 - (938\ \text{MeV})^2}} = 4.22\times10^{-6}\text{nm} = 4.22\ \text{fm}$$

or

$$\lambda = \frac{hc}{\sqrt{K(K + 2E_0)}} = \frac{(1240\ \text{eV}\cdot\text{nm})(1\ \text{MeV}/10^6\ \text{eV})}{\sqrt{45\ \text{MeV}(45\ \text{MeV}+2(938\ \text{MeV}))}} = 4.2\times10^{-6}\text{nm} = 4.2\ \text{fm}$$

Proton KE = 2.0 MeV

$$\lambda = \frac{hc}{\sqrt{E^2 - E_0^2}} = \frac{(1240\ \text{eV}\cdot\text{nm})(1\ \text{MeV}/10^6\ \text{eV})}{\sqrt{(940\ \text{MeV})^2 - (938\ \text{MeV})^2}} = 2.02\times10^{-5}\text{nm} = 20.2\ \text{fm}$$

or

$$\lambda = \frac{hc}{\sqrt{K(K + 2E_0)}} = \frac{(1240\ \text{eV}\cdot\text{nm})(1\ \text{MeV}/10^6\ \text{eV})}{\sqrt{2.0\ \text{MeV}(2.0\ \text{MeV}+2(938\ \text{MeV}))}} = 2.0\times10^{-5}\text{nm} = 20\ \text{fm}$$

Electron 3% speed of light, $v = 0.03c$

$$\gamma_p = \frac{1}{\sqrt{1-\left(\frac{0.03c}{c}\right)^2}} = 1.00045$$

$$E = \gamma_p mc^2 = (1.00045)(0.511\ \text{MeV}) = 0.5112301\ \text{MeV}$$

K<<E.

$$\lambda = \frac{hc}{\sqrt{E^2 - E_0^2}} = \frac{(1240\ \text{eV}\cdot\text{nm})(1\ \text{MeV}/10^6\ \text{eV})}{\sqrt{(0.5112301\ \text{MeV})^2 - (0.511\ \text{MeV})^2}} = 0.081\ \text{nm} = 8.1\times10^4\ \text{fm}$$

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