Part I: Classical Probability with Dice and Coins

In general, the probability for a particular outcome is the **ratio** of the number of desired outcomes to the total number of possible outcomes.

1. If we roll a six-sided die, what is the probability that we obtain a 1?

   $P_1 =$

2. If we roll a six-sided die, what is the probability that we obtain a 3?

   $P_3 =$

3. When tossing a die, what is the probability of rolling either a 1 **or** a 3?

   $P_{1 \text{ or } 3} =$

4. How is $P_{1 \text{ or } 3}$ related to P_1 and P_3?

5. Imagine simultaneously tossing a die and flipping a coin. How many possible outcomes (combinations of a certain number and either heads or tails) are there?

6. What is the probability of rolling a 2 **and** getting tails?

   $P_{2 \text{ and tails}} =$

7. How is $P_{2 \text{ and tails}}$ related to P_2 and P_{tails}?

8. Complete the following sentence by selecting the appropriate words in parentheses:

   In general, the word **OR** is a signal to (add/subtract/multiply/divide) the individual probabilities, and the word **AND** is a signal to (add/subtract/multiply divide) the individual probabilities.

   Note the aforementioned sentence is only true if the individual probabilities are *independent* of one another!

9. Imagine flipping a coin three times. What is the probability of obtaining heads twice? To aid in answering this question, fill out the following table, listing all the possible outcomes. The first two columns of the first trial has been filled out for you, indicating the outcome when the first, second, and third flip are all heads.

Outcome	# Heads	Probability
HHH	3	

10. List the probabilities of obtaining three heads, two heads, one head, and zero heads. Find their sum.

$P_{3\text{ heads}} =$

$P_{2\text{ heads}} =$

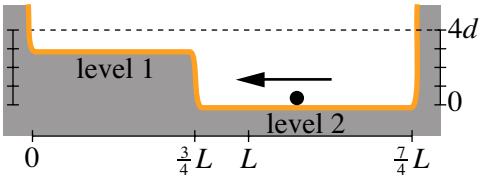
$P_{1\text{ head}} =$

$P_{0\text{ heads}} =$

$$\sum_i P_i = P_{3\text{ heads}} + P_{1\text{ heads}} + P_{1\text{ head}} + P_{0\text{ heads}} =$$

11. Imagine tossing two dice simultaneously. What is the probability that the sum of the results is four?

Part II: Classical Probability, Ball on Track



A ball rolls back and forth on a track with very steep sides. Two levels of unequal length are joined by a steep ramp to form the base of the track. Assume that the time spent on the steep portions is negligible, that there is no friction in the system, and that the ball rolls smoothly, without bouncing, up and down, forever back and forth between Level 1 and Level 2. Level 1 has a length of $3L/4$, and level 2 has a length of L. Assume the ball was dropped from a position of $4d$.

1. Is the speed of the ball on level 1 *greater than, less than,* or *equal to* its speed on level 2? Explain.

2. Assume the speed of the ball on level 1 is v_1 and the speed of the ball on level 2 is v_2. Determine the ratio v_2/v_1. Find a constant number; no other variables should appear in your answer. From here on, keep answers exact (use fractions, not rounded decimals).

3. Is the amount of time that the ball spends on level 1 *greater than, less than,* or *equal to* the amount of time it spends on level 2? Explain.

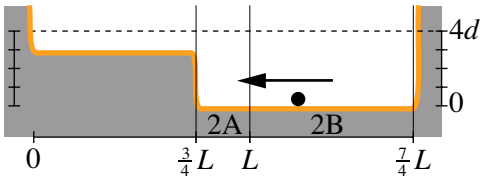
4. Suppose a single photograph were taken at a random time. On the basis of your results above, is the probability of the photograph showing the ball on level 1 *greater than*, *less than*, or *equal to* that of the photograph showing the ball on level 2? Explain.

5. Determine the probability of finding the ball on each level. Explain.

✓ Check your results with your instructor.

Part III: Probability Density

Imagine splitting level 2 into two unequal segments: segment "2A" from $x = 0.75L$ to $x = L$, and segment "2B" from $x = L$ to $x = 1.75L$.



1. Find the probability that the ball is found in the indicated region (out of anywhere in the system). Explain how you got your answer.
   a. Segment 2A:

   b. Segment 2B:

Consider the following ratio for each segment: the probability of finding the ball within that segment divided by the length of the segment.

2. Is the above ratio for segment 2A *greater than, less than,* or *equal to* the above ratio for segment 2B? Explain.

The ratio defined above is called *probability density*.

3. In your study of physics, you may have encountered different kinds of density. For example, (linear) charge density may be interpreted as the amount of charge in a region of unit length. How are charge density and probability density similar?

4. Consider the student discussion below:

   Student 1: "If the probability in one region is larger than another region, then the probability density must also be larger."

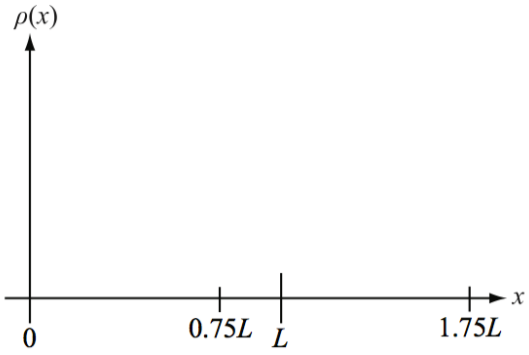
   Student 2: "I disagree. We are dividing by length, so a smaller region will always have a larger probability density."

   Student 3: "Both of you are partially right. I can only really compare the probability density in two regions if I know both the probability and the length of the two regions."

   With which student do you agree, if any? Explain.

✓ Check your results with your instructor.

Part IV: Probability density graphs



1. In the space at right, carefully draw a graph of probability density, $\rho(x)$, versus position from $x = 0$ to $x = 1.75L$. Label relevant values on the vertical axis.

2. How would you determine the probability of finding the ball between $x = L/2$ and $x = L$? Explain.

3. What is the probability of finding the ball *exactly* at $x = L$? Explain.

4. What answer would you expect for the probability of finding the ball between $x = 0$ and $x = 1.75L$? Show that your graph of probability density gives you the answer you expect.

5. Suppose you were given an arbitrary probability density function $\rho(x)$ (i.e., one that does not have a shape as simple as the one above). Write a mathematical expression for the probability of finding the ball between $x = x_1$ and $x = x_2$.

6. Do you agree or disagree with the statement below? Explain.
   "The probability densities have to add up to one, just like the probabilities, because it has to be somewhere."

✓ Check your results with your instructor.