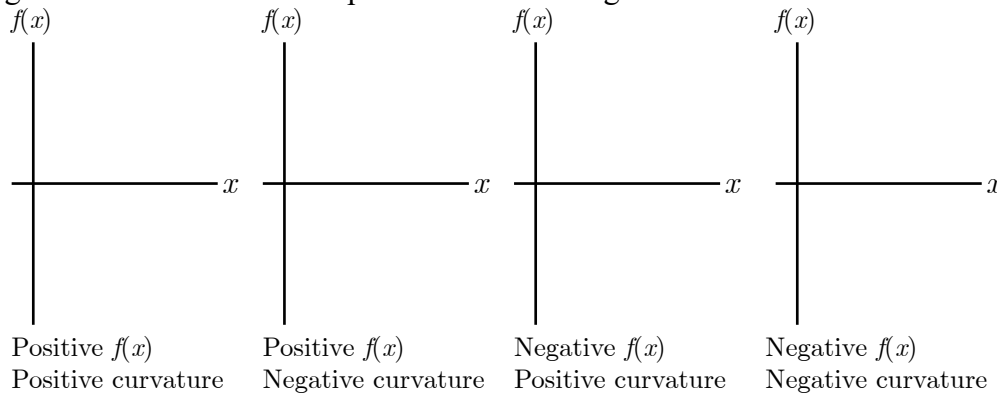


Part I: The Shape of the Wave Function

For a state with energy E_n in a system with potential $V(x)$, the wave function $\psi_n(x)$ satisfies the time-independent Schrödinger equation:

$$-\frac{d^2\psi_n(x)}{dx^2} = -\frac{2m}{\hbar^2}[E_n - V(x)]\psi_n(x)$$

1. The left-hand side of the Schrödinger equation is the *curvature* of $\psi_n(x)$. Draw some segments of functions that represent the following combinations of values and curvatures.



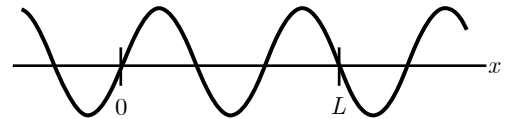
2. Which combinations produce functions curved *toward* the axis? Which combinations produce functions curved *away* from the axis? Which combinations could be wave-like?
3. Consider a region where the total energy E_n is *less than* the potential energy $V(x)$. What is the general shape of the function $\psi_n(x)$ in such a region? (*i.e.*, is the wave function linear, exponential, sinusoidal, quadratic, etc.?) Explain.
4. Now consider a region where the total energy is *equal* to the potential energy throughout the region. What is the general shape of the function, $\psi_n(x)$, in such a region?
5. Now consider a region where the total energy is *greater than* the potential energy? What is the general shape of the function, $\psi_n(x)$, in such a region?

Part II: Interpreting the Wavenumber of a Sinusoidal Wavefunction

In those region(s) where the wave function is sinusoidal, it may be expressed in the form, $\psi_n(x) = A\cos(kx) + B\sin(kx)$. The constant k is often called the *wave number*.

1. How is k related to the quantities in the equation at the top of the previous page? (*Hint: Insert the form given for $\psi_n(x)$ into the time-independent Schrödinger equation and solve for k .*)
2. Reminder yourself how k is related to the wavelength of a sinusoidal wave.

3. Find k in terms of L for the wave shown at right. Sketch waves with wave numbers of $2k$ and $k/2$.



4. Consider the student discussion below.

Student 1: "The wave number is proportional to energy. The more oscillations per unit length, the more energy the wave has."

Student 2: "I think k is proportional to the difference between total energy and potential energy. So higher wave number means higher *kinetic energy*, not just energy."

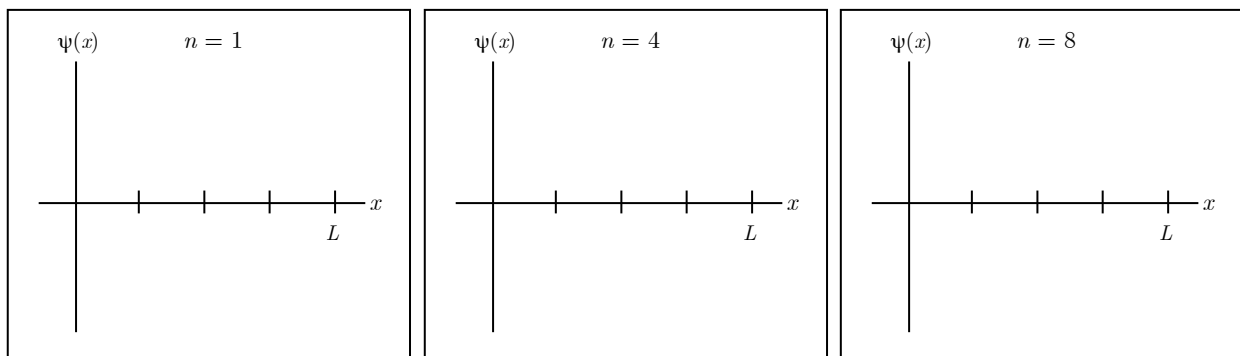
With which student do you agree, if either? Explain.

Part III: The Particle in a Box (a.k.a. The Infinite Square Well Potential)

You've already seen that for the boundary conditions of a particle confined to exist between 0 and L , with zero potential energy between 0 and L and infinite potential energy outside of this region, the solution to the time-independent Schrödinger equation is a sinusoidal function. You've also already found the normalization constant (the amplitude of the function).

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

1. Sketch the function, $\psi_n(x)$ that corresponds to a particle in each of the energy states below. Label the vertical axis with the appropriate values for each case.



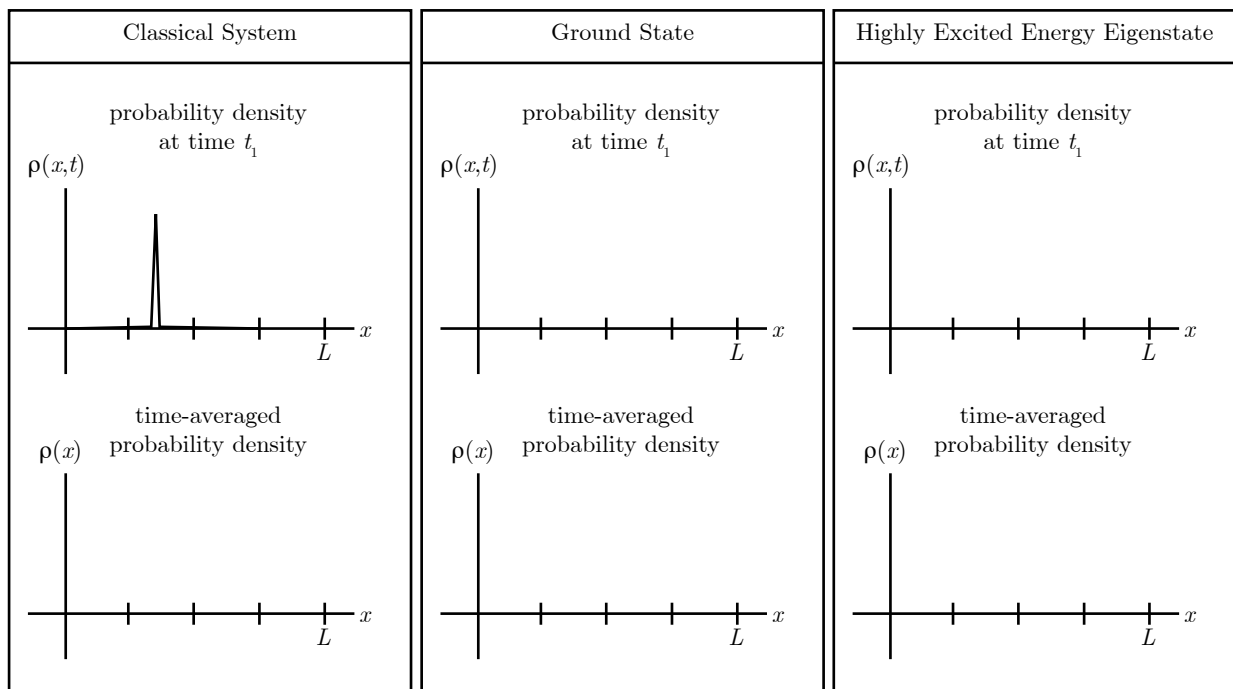
2. Count how many “bumps” appear in each sketch. (More mathematically speaking, this is the number of antinodes.) What is the general rule to determine the number of “bumps” to draw given an energy state n ?
3. Find an expression for the total energy of each state, E_n . (*Hint*: use your answer from the first question of Part II on the previous page!)
4. Interpret the solution.
 - a. Is it quantized? Why? How does it vary with n ?
 - b. Can the particle in the box have total energy of zero? Can it be at rest? Explain in terms of the Heisenberg uncertainty principle.

5. Write the full wave equation, $\psi_n(x)$ for these states, in terms of E_n .

6. Find the equation for the probability density as a function of position and time, $\rho(x, t)$, for these states.

Part IV: The Correspondence Principle

1. Consider the quantum mechanical infinite square well, as well as its classical analogue. For each of the systems below complete the graphs of (1) the probability density at some arbitrary time, and (2) the time-averaged probability density.



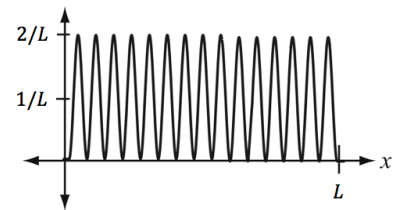
2. Suppose that you have been given a large ensemble of *identically prepared* systems corresponding to one of the three cases above (*i.e.*, each corresponds to the same case).
 - a. If you were to measure the position of each system at *the same time*, t_1 , could you determine to which of the three cases above your ensemble corresponds? Explain. (*Hint*: How do the graphs above relate to the probabilities of measurements of position?)

- b. If, instead, you were to measure the position of each system at *a random time*, could you determine to which of the three cases above your ensemble corresponds? Explain.
3. Suppose you were to decrease the spatial resolution of your detector (*i.e.*, it makes *less* precise measurements). Would any of your answers to Part B change? Explain.

Quantum mechanics was developed in order to account for phenomena that classical mechanics could not. If it is to be regarded as a complete theory, it must also be able to account for systems that classical mechanics can describe. The *correspondence principle* states that the time-averaged probability density for a highly-excited quantum mechanical system should be experimentally indistinguishable from that of the analogous classical system.

1. Consider the following discussion between two students:

Student 1: "The correspondence principle says that the quantum probability density approaches the classical one for highly-excited systems. I've drawn the probability density for a quantum system with large n . So the time-averaged probability density for a classical particle in an infinite square well is equal to $2/L$."



Student 2: "If you calculate the area under the classical probability density of $2/a$ it equals $2!$ So in order to be normalized, it must be at $1/a$ instead. So the correspondence principle is violated in this case."

Neither student is correct. Identify the errors made by each student.