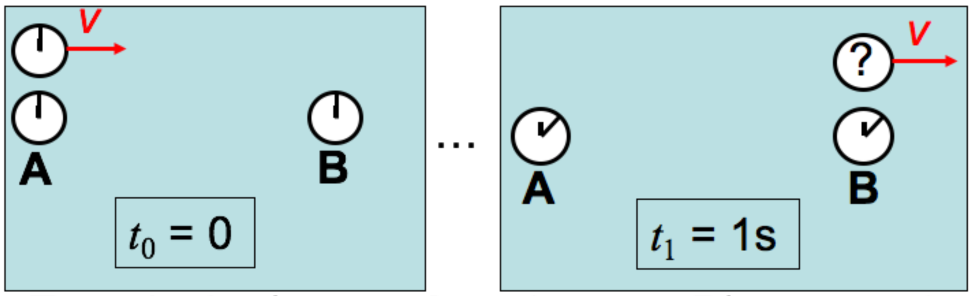
Warm-Up Problem: Lorentz Transformations



$$t' = \gamma(t - \frac{v}{c^2}x)$$

$$t = \gamma(t' + \frac{v}{c^2}x')$$

Two clocks (one at A and one at B) are synchronized. A third clock flies past A at a velocity v. The moment it passes A all three clocks show the same time $t_0 = 0$ (viewed by observers in A and B. See left image.)

What time does the third clock show (as seen by an observer at B) at the moment it passes the clock in B? The clock at B is showing $t_1 = 1$s at that moment. *Use the Lorentz transformation, and simplify!*

A) $\gamma(t_1 - t_0)$ B) $\gamma^2(t_1 - t_0)(1 - v/c^2)$ C) $\gamma^2(t_1 - t_0)(1 + v^2/c^2)$

D) $(t_1 - t_0)/\gamma$ E) $\gamma(t_1 - t_0)(1 + vx'/c^2)$

Part I: Classical vs. Relativistic Momentum

An electron has a mass $m \approx 9.11 \times 10^{-31}\,\text{kg}$. Fill in the table below to show the classical and relativistic momentum of the electron at various speeds.

u	γ_p	Classical $p_c = mu \left[10^{-22}\,\frac{\text{kg}\cdot\text{m}}{\text{s}}\right]$	Relativistic $p = \gamma_p mu \left[10^{-22}\,\frac{\text{kg}\cdot\text{m}}{\text{s}}\right]$	% Difference $\frac{\lvert p_c - p\rvert}{p_c}$
0.1c				
0.5c				
0.9c				
0.99c				

Part II: Velocity as a Function of Time with Constant Applied Force

Restricting ourselves to one dimension, Newton's Second Law can be expressed as $F_x = \frac{dp_x}{dt}$. Imagine a particle with mass *m* that is at rest at position $x = 0$ and time $t = 0$. At that moment, a constant force *F* starts being applied to the particle.

1. Using the *classical* definition of momentum $p = mu$, find an expression for the velocity as a function of time. (Hint: Multiply both sides of Newton's Second Law by *dt*, and integate both sides.)

2. Describe the mathematical form of this relationship between velocity and time. Is this what you expected from mechanics?

3. Using the *relativistic* definition of momentum, $p = \gamma_p mu$ where $\gamma_p = \frac{1}{\sqrt{1-u^2/c^2}}$, find an expression for the velocity as a function of time. Follow your procedure for the classical definition, but after integration, write the full expression for *p* and γ_p in terms of *u*, and gather all *u* terms to one side.

4. What is the shape of this graph of velocity vs. time? It is difficult to see by eye. Open a browser to www.wolframalpha.com. Let's define the speed of light $c = 1$ so that all velocities are expressed as a function of the speed of light. Choose arbitrary values for F and m as well. Type "plot [your function] from $t = 0$ to 4" (for example). Draw a sketch of the result here.

5. What can you conclude from reading this graph?

Part III: Relativistic Kinetic Energy

Use the binomial approximation $(1+x)^n \approx 1+nx$ when $x << 1$ to approximate the relativistic kinetic energy $K = (\gamma_p - 1)mc^2$ when $v << c$.