**Conservation of Energy-Momentum in Special Relativity**

*This problem is to be solved without a calculator!*

Imagine that somewhere in deep space a rock with mass $m_1$ = 12 kg is moving in the +*x* direction with $u_{1x}$ = (4/5) *c* in some inertial frame. This rock strikes another rock of mass $m_2$ = 28 kg at rest ($u_{2x}$ = 0). Pretend that the first rock, instead of instantly vaporizing into a cloud of gas (as any *real* rocks colliding at this speed would), simply bounces off the more massive rock and is subsequently observed to have an *x*-velocity $u_{3x}$ = (–5/13) *c*. What is the *x*-velocity $u_{4x}$ of the larger rock after the collision?

1. Draw a picture of the "before" and "after." Label all relevant quantities.

2. Calculate (by hand) $\gamma_{p1}$, $\gamma_{p2}$, and $\gamma_{p3}$.

Conservation of energy-momentum tells us that:
$$\begin{bmatrix} E_1 \\ p_{1x} \\ p_{1y} \\ p_{1z} \end{bmatrix} + \begin{bmatrix} E_2 \\ p_{2x} \\ p_{2y} \\ p_{2z} \end{bmatrix} = \begin{bmatrix} E_3 \\ p_{3x} \\ p_{3y} \\ p_{3z} \end{bmatrix} + \begin{bmatrix} E_4 \\ p_{4x} \\ p_{4y} \\ p_{4z} \end{bmatrix}$$

3. Use the expressions for total energy of a particle and momentum in each direction to write two equations (one for conservation of total energy, one for conservation of *x*-momentum), which will include multiple unknowns $m_4$, $\gamma_{p4}$, and $u_{4x}$.

4. Use your equations to solve for $u_{4x}$. Do this entirely in terms of variables; no numbers yet. Simplify as much as possible.

5. Now plug in numbers. You should be able to do this math by hand. No calculators!

6. (If there is time.) Show that the mass of the large rock is unchanged.

7. (If there is time.) Show that this example *does not* conserve Newtonian momentum.