Part I: Wave Velocity and Graphing Tools

A transverse wave pulse, described by

$$y(x,0)=\frac{4}{x^2+2}$$

is initiated at t = 0 in a stretched string.

1. I seem to have forgotten to include the units with the two numbers. What units does the 4 have, and what units does the 2 have? Assume lengths are in meters.

2. Write an equation for the traveling pulse if it moves with a speed of 2.5 m/s in the negative x-direction. **Hint:** Change the argument of the function from "x" to "$x \pm vt$".

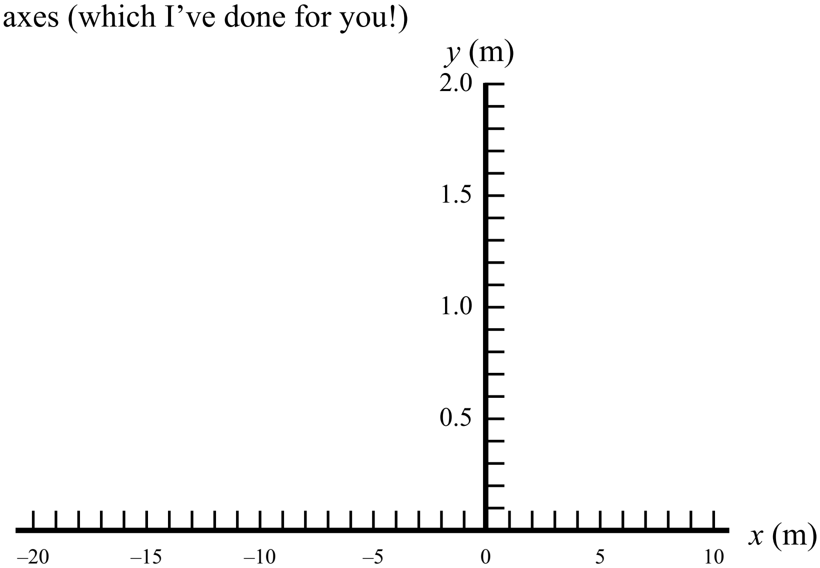
3. WolframAlpha is a helpful tool to allow you to visualize waveforms. It is the online version of a more powerful program called Mathematica, which is available on Westminster Anywhere. Open WolframAlpha.com in a web browser, type in the following, then press enter:

```
plot 4/(x^2+2)
```

If we don't specify the desired plot range, WolframAlpha will assume a range. But, if we want to compare this waveform at different times, it'd be nice to keep the ranges of the x and y axes consistent from plot to plot. Now type in the following and press enter:

```
plot 4/(x^2+2), x=-20 to 10, y=0 to 2
```

Sketch the shape of the wave pulse at time t = 0 s below. Notice that WolframAlpha doesn't know the units of the constants or what these variables represent; it's up to you to properly label your coordinate axes (which I've done for you!)



4. Is the snapshot of the wave pulse at time $t = 0$ s an odd or even function?

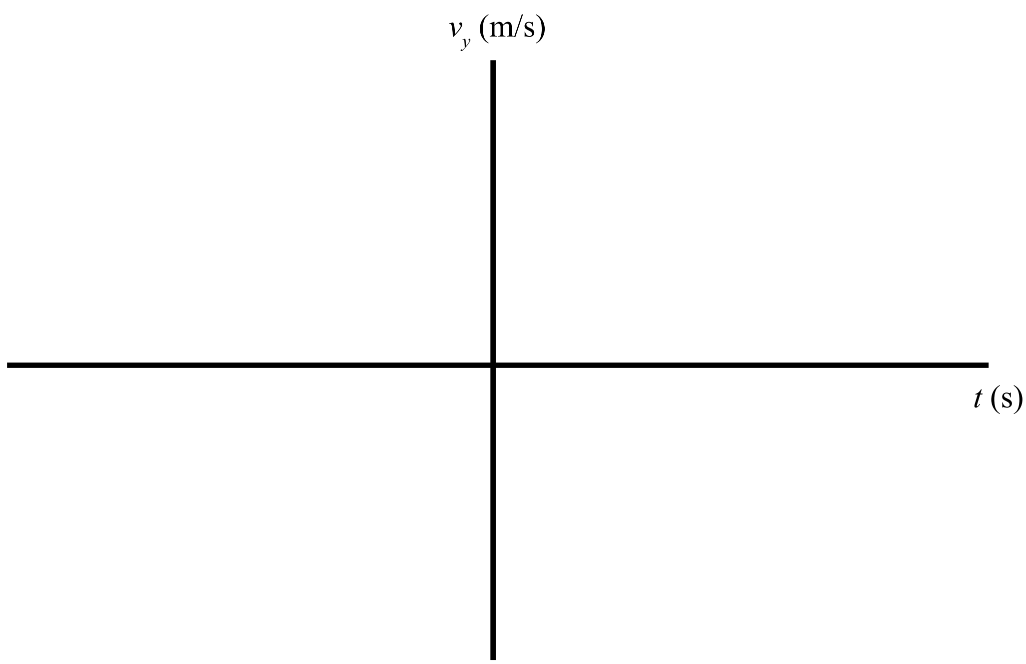
5. Using Wolfram Alpha and your equation which includes the variable t, sketch the pulse at $t = 2$ s and $t = 5$ s on your original graph. Be sure to label the different curves.

6. Confirm, using your graph, that the wave velocity is in fact $v_x^{wave} = -2.5\,\mathrm{m/s}$. Describe your method.

Part II: Particle Velocities

Though the wave pulse moves from right to left, the individual particles in the string move in the vertical direction, up and down.

7. In the space below, predict the general shape of the graph of y-velocity vs. time for the particle of string at $x = 0$. *Hints:* do you predict this will be an odd or even function, based on your answer to Question 4 above? At times after t = 0 s, is the particle on the string at $x = 0$ m moving up or down? What about before time $t = 0$ s?



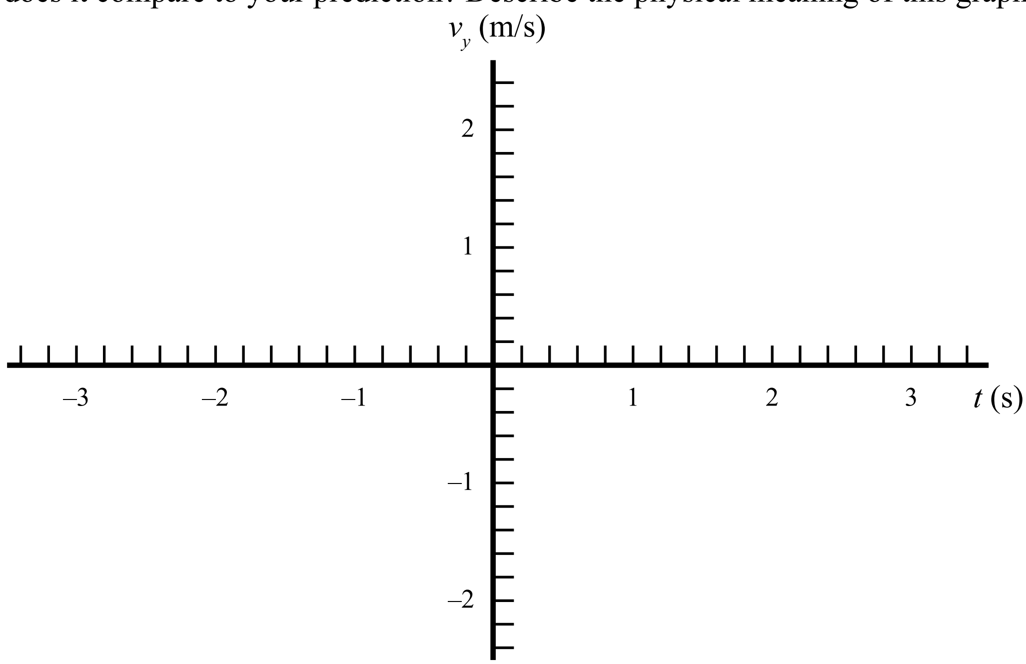
Reminder! Don't move on until your whole group has discussed your predictions and reasoning.

The instantaneous velocity is defined as the derivative of position with respect to time. In our case, we will be using a partial derivative, because the y-position is a function of both x-position and time. When taking a partial derivative with respect to time, you would consider the position along the string, x, to be constant.

$$v_y = \frac{\partial y(x,t)}{\partial t}$$

8. Use the above equation to find the y-velocity of the particles of the wave pulse as a function of time. You will have to use the chain rule and may also use the quotient rule.

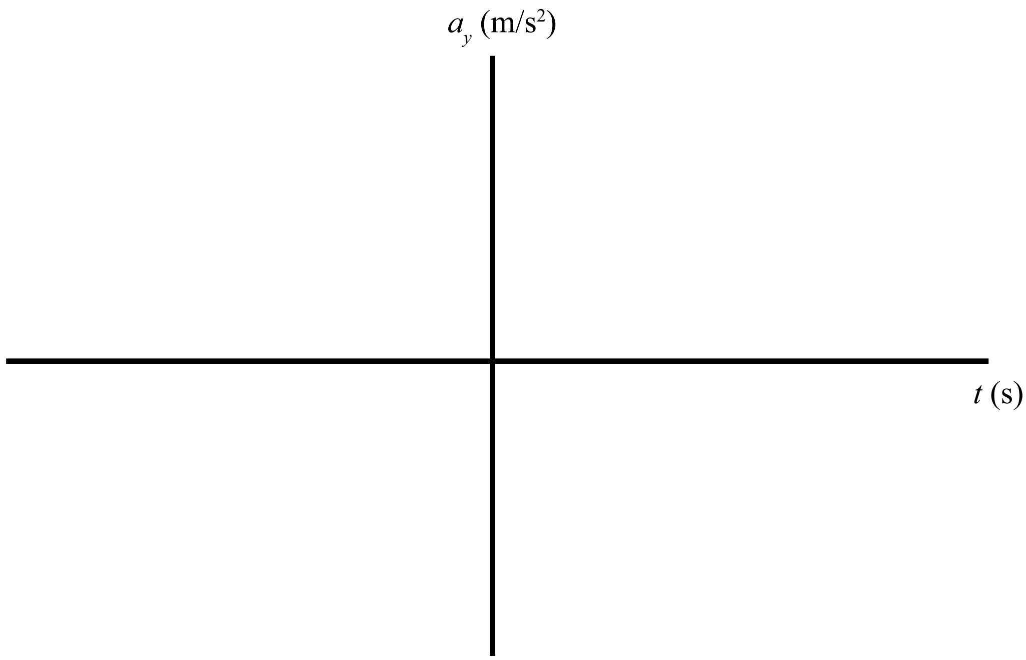
9. Use WolframAlpha to plot v_y as a function of time when $x = 0$ m. Draw this graph below. How well does it compare to your prediction? Describe the physical meaning of this graph in words.



In order to talk about forces and energy, we'll want to think about acceleration first. Acceleration is the second derivative of position with respect to time.

$$a_y = \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial v_y(x,t)}{\partial t}$$

10. In the space below, predict the shape of the graph of y-acceleration vs. time for the particle of string at $x = 0$ m.

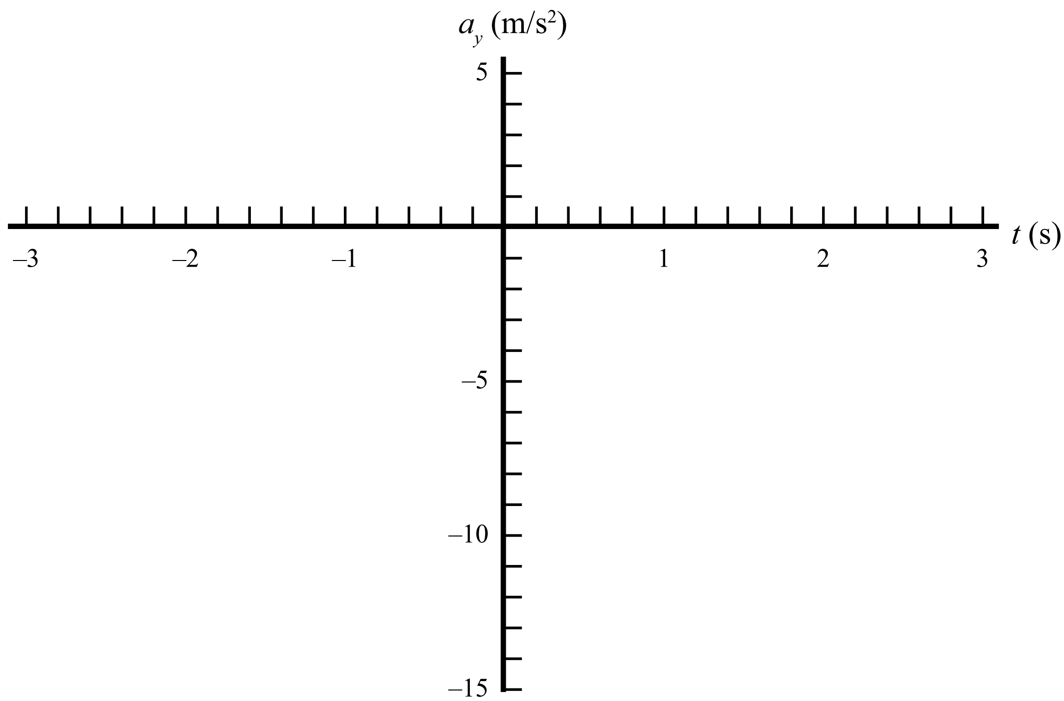


11. You are capable of using the above equation to find a_y of a particle on the string as a function of time by hand. However, for the sake of time in class, let's ask WolframAlpha to help us out. Type the following into WolframAlpha:

```
d^2/dt^2 4/((x+2.5t)^2+2)
```

Write the result below *and include units on all constants*. Yes, it's annoying and tedious! Life is often annoying and tedious, it's not all sinusoids and rainbows out there!

12. Use WolframAlpha to plot a_y as a function of time when $x = 0$ m. Note that if you click on the result from the previous calculation, WolframAlpha will use that output as a new input, so you don't have to type the whole thing by hand. However, you may have to edit the command slightly to get your plot range to match the template below. Also make sure the leading "4" is in fact multiplied by the rest of the expression. Draw the graph below.



13. Describe the physical meaning of this graph in words. What happens as the wave pulse approaches, meets, and moves away from the little bit of string located at $x = 0$ m?

14. When is the magnitude of the force on the particle of string at $x = 0$ m greatest? Describe the particle's y-velocity and displacement at this time.

15. (If time.) Predict what a graph of a_y *vs.* t would look like for $x = -5$ m (just describe, no need to sketch). Try it using WolframAlpha; what happens?