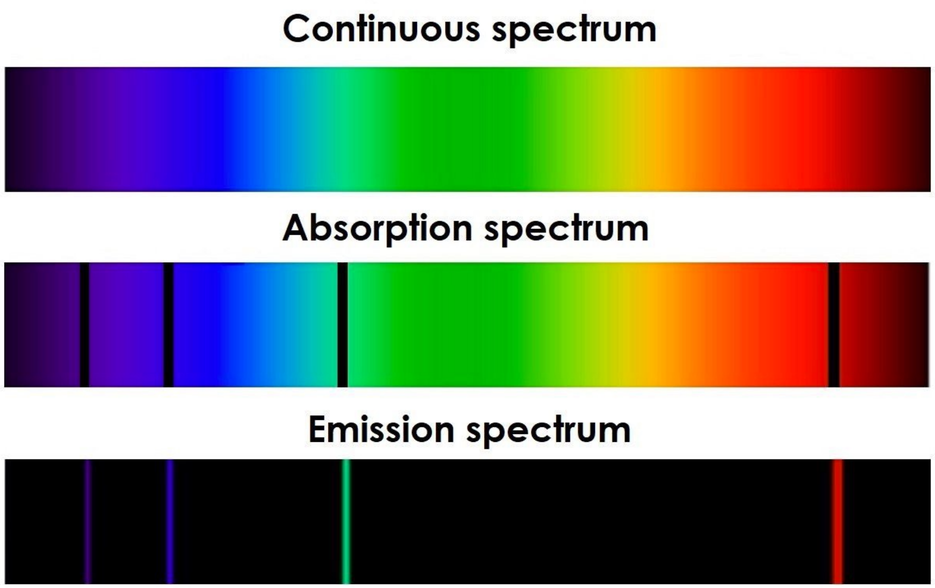
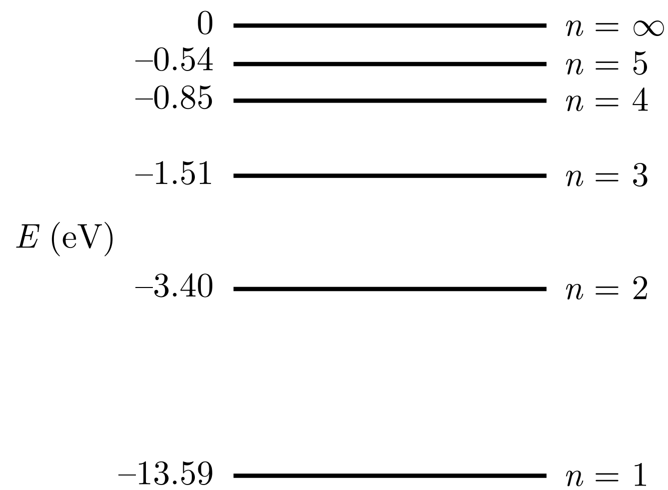
Phys 301 Class 20
De Broglie Wavelength
Matter Waves

Some slides from Dr. Charles Bailey at
Colorado School of Mines

Clarification

- Photon absorption is “all or nothing”





Successes of Bohr Model

- 'Explains' source of Balmer formula and predicts empirical constant R_H (Rydberg constant) from fundamental constants.
- Explains why R_H is different for different single electron atoms (called *hydrogen-like ions*).
- Predicts approximate size of hydrogen atom.
- Explains (sort of) why atoms emit discrete spectral lines.
- Explains (sort of) why electron doesn’t spiral into nucleus.

Shortcomings of the Bohr model:

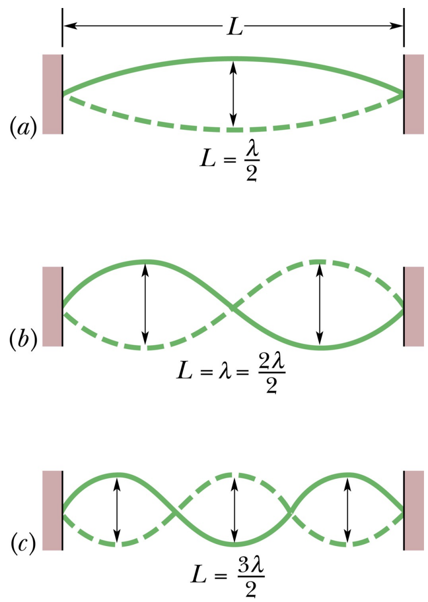
- Why is angular momentum quantized yet Newton’s laws still work?
- Why don’t electrons radiate when they are in fixed orbitals yet Coulomb’s law still works? (Accelerating charges radiate energy.)
- No way to know *a priori* which rules to keep and which to throw out…
- Can't explain shapes of molecular orbitals and how bonds work.
- Can’t explain doublet spectral lines (Zeeman effect)

De Broglie hypothesis (1924)

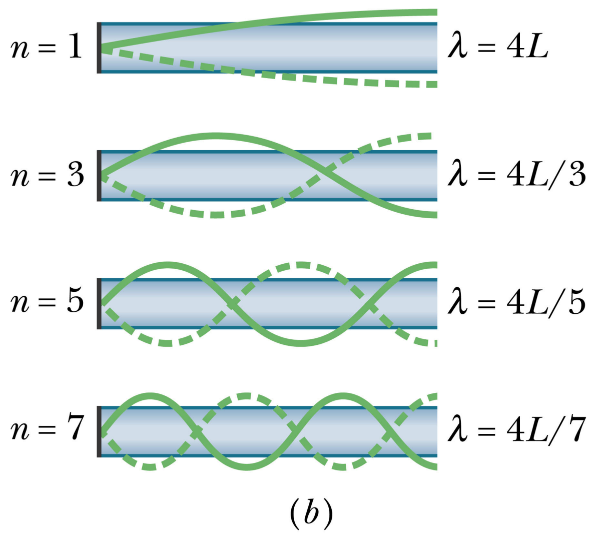
- If a light wave is *also* a particle (photon)…
  - What is the experimental evidence for these two statements?
- Could particles (electrons, protons, all forms of matter) also be WAVES?
  - How could we test this?

Recall Standing Waves…

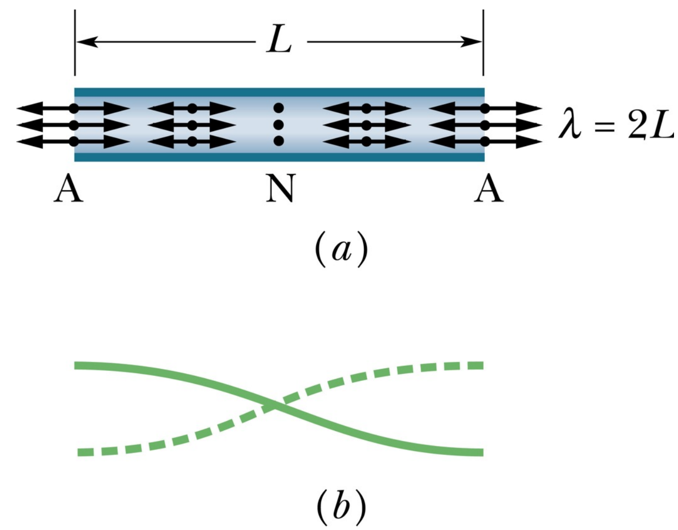
- "Rules" depend on Boundary Conditions.



Two Fixed Boundaries



One Fixed, One Open Boundary



Two Open Boundaries

Standing Waves on a Ring

Just like standing wave on a string, but now the two ends of the string are joined.

What are the restrictions on the wavelength?

A. $r = \lambda$

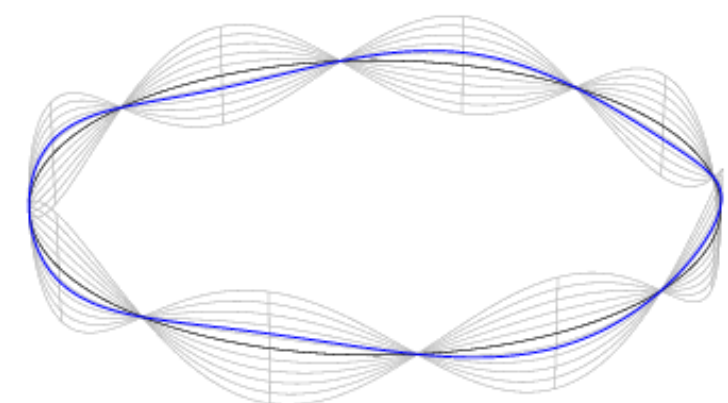
B. $r = n\lambda$

C. $\pi r = n\lambda$

D. $2\pi r = n\lambda$

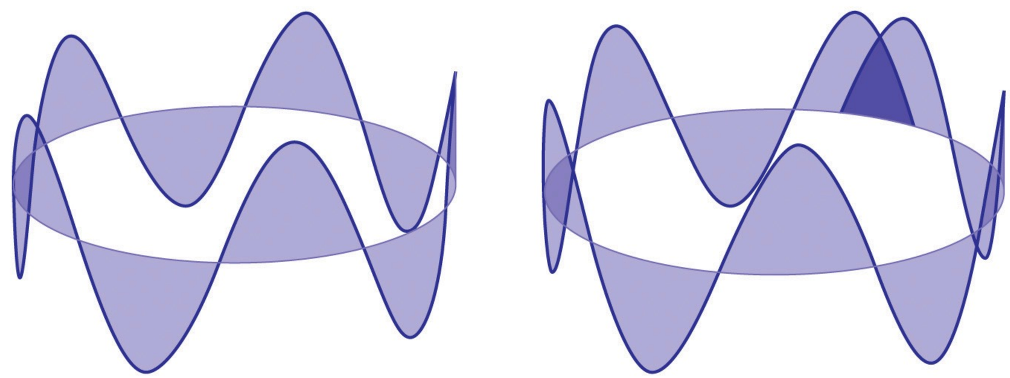
E. $2\pi r = \lambda/n$

$n = 1, 2, 3, \ldots$



Standing Waves on a Ring

- Answer: D. $2\pi r = n\lambda$



(a) Standing wave

(b) Destructive Interference

- Circumference = $2\pi r$
- To get standing wave on ring:

  Circumference = $n\lambda$

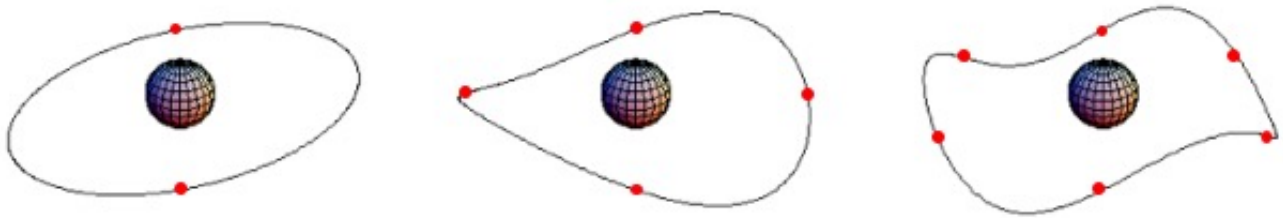
  Must have integer number of wavelengths to get constructive, not destructive, interference.
- n = number of wavelengths

Seems promising…

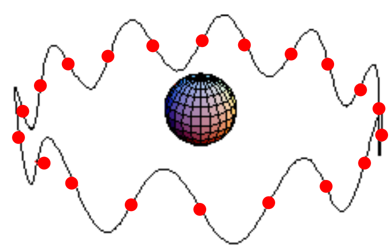
- Bohr model requires quantized energy of particle…
- … and standing waves in a ring would have quantized wavelength.

deBroglie Waves

$n = 1$ $\quad$ $n = 2$ $\quad$ $n = 3$



… $n = 10$



• = node = fixed point that doesn’t move.

Recall from Special Relativity…

- Despite having no mass, a photon of light still has energy and momentum.

$$E = hf = \frac{hc}{\lambda}$$

$$E_0^2 = E^2 - (pc)^2$$

If mass = 0, $E_0 = mc^2 = 0$

$$E = pc$$

$$p = \frac{E}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda}$$

- So, what if particles with momentum p also have a wavelength?

deBroglie Waves

- Substituting the deBroglie wavelength ($\lambda = h/p$) into the condition for standing waves ($2\pi r = n\lambda$), gives:

$$2\pi r = nh/p$$

- Or, rearranging:

$$rp = nh/2\pi$$
$$L = n\hbar$$

- deBroglie EXPLAINS quantization of angular momentum, and therefore EXPLAINS quantization of energy!

In the deBroglie picture, the electrons have an intrinsic wavelength associated with them. We have also been told that one wavelength fits around the circumference for the n =1 level of hydrogen, 2 fit around the circumference for $n = 2$, 5 fit for $n = 5$, etc.

Therefore, we expect that the $n = 5$ circumference is 5 times as large as the $n = 1$ circumference.

A) True
B) False

False!

From the Bohr model we know that:

$$r_n = n^2 a_0$$

Here the other thing changing with n is the deBroglie wavelength of the electrons, because the electron energy and momentum also change with n.

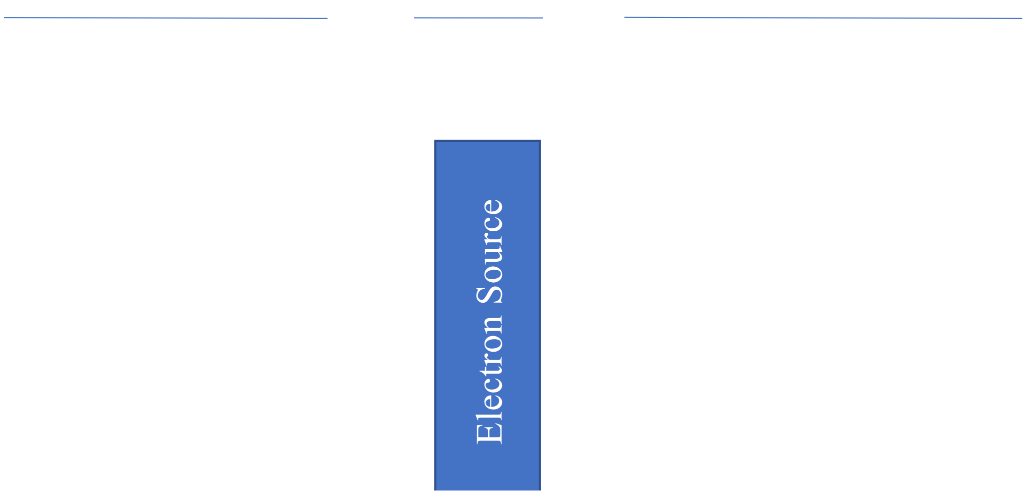
Handout A
Part I: Expressions for de Broglie Wavelength

It works, but…

- Show me the MONEY! I mean, EXPERIMENTAL EVIDENCE!
- 3 such experiments:
  - Davisson-Germer, 1927 (de Broglie wavelength)
  - Two-Slit Interference of Electrons
    - Jönsson 1961 (beam of electrons)
    - Pozzi 1974, Tonomura 1989 (single electrons)

Two-Slit Interference with Electrons

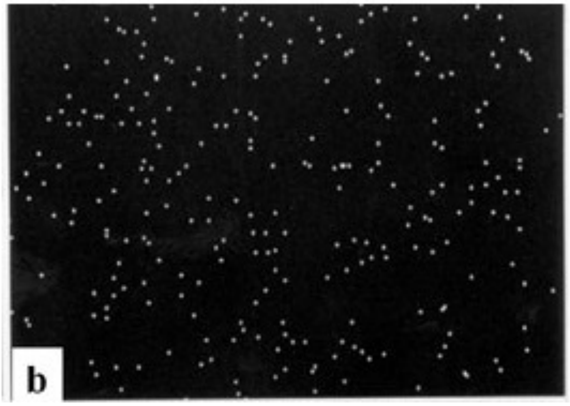
8 electrons



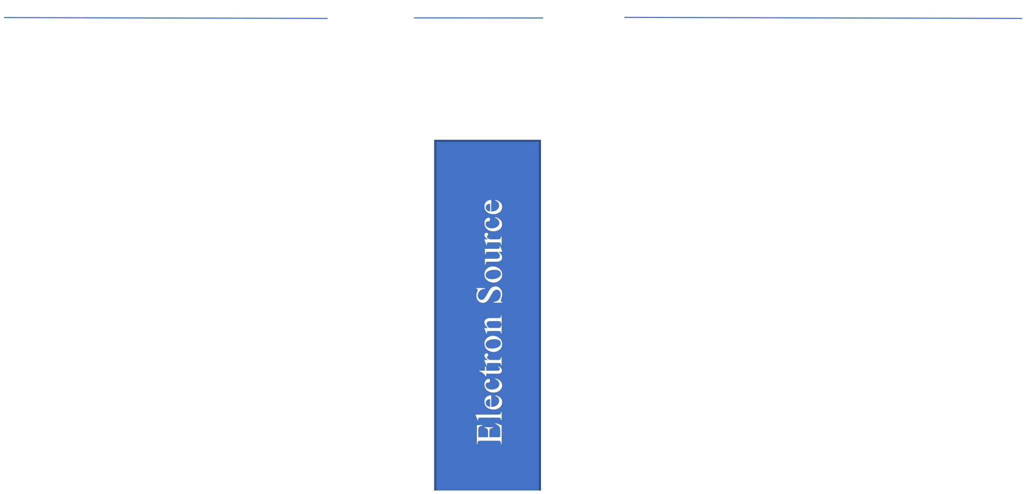
- Electrons fired one at a time.

Two-Slit Interference with Electrons

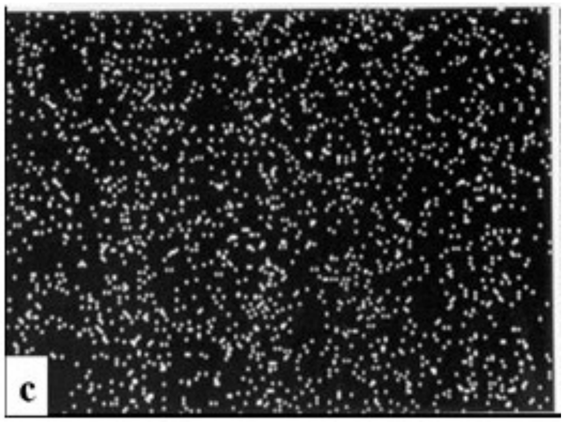
- Electrons fired one at a time.



270 electrons

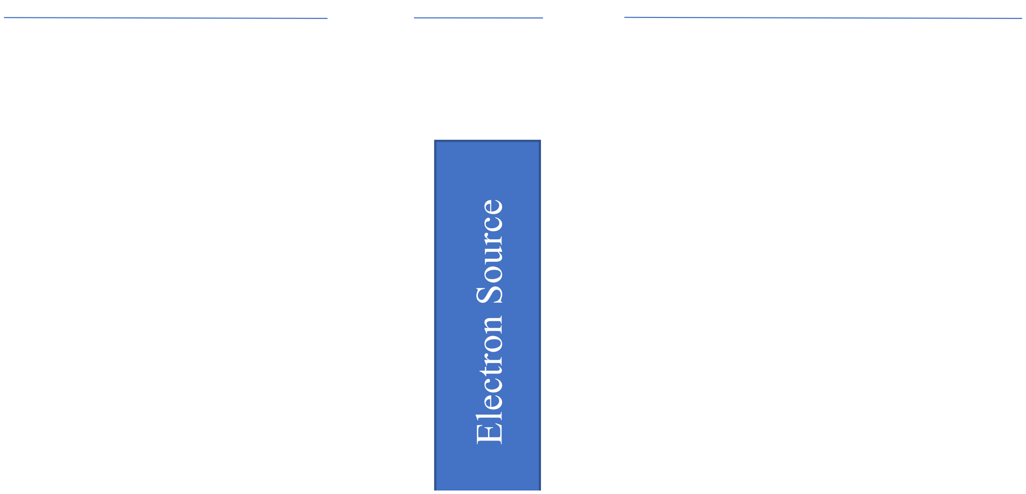


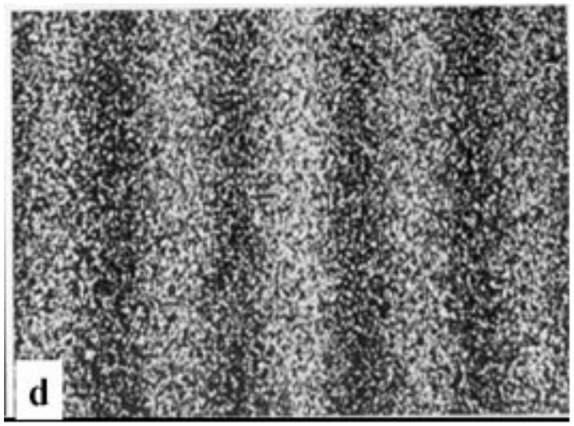
Two-Slit Interference with Electrons



2000 electrons

- Electrons fired one at a time.

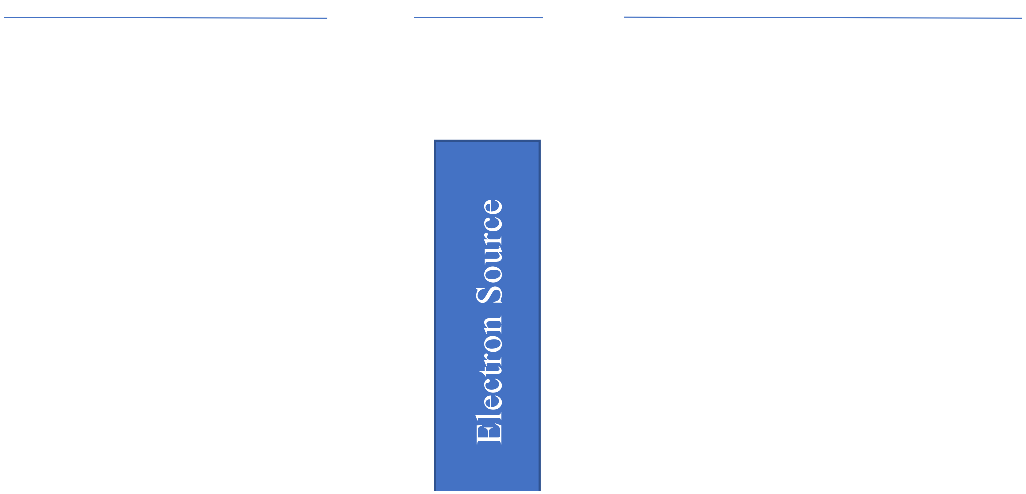




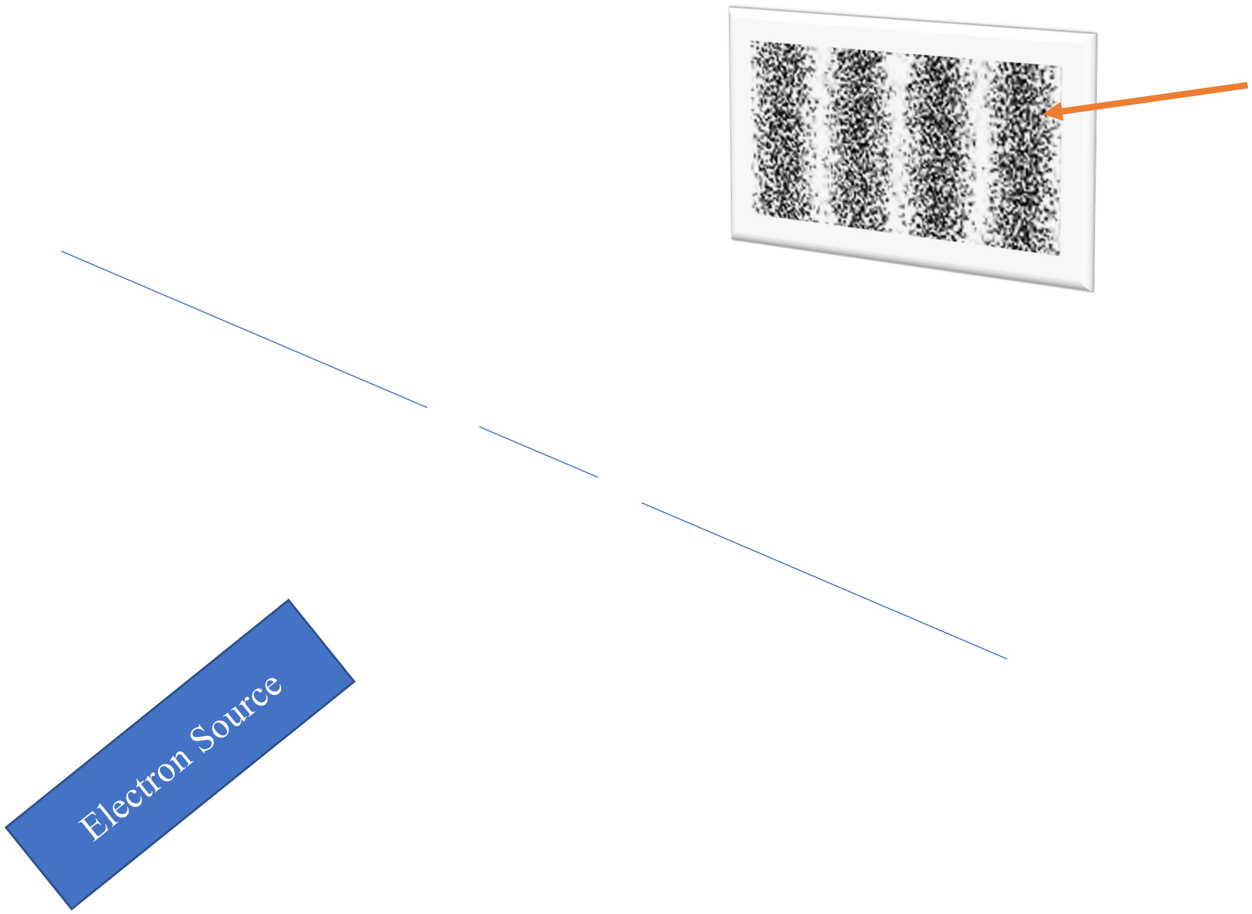
160,000 electrons

Two-Slit Interference with Electrons

- Electrons fired one at a time.



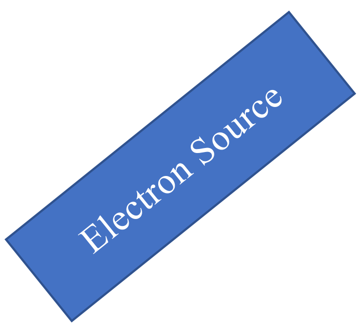
Handout: Wave Properties of Matter



Which slit did THIS electron pass through?

A. Left Slit
B. Right Slit
C. Either Left or Right, but Cannot Tell
D. Both Slits

Each electron passes through **both slits**, interferes with itself, then becomes **localized when detected.**



The pattern that emerges means there’s a higher **probability** of detecting an electron in some locations than others.

What does it MEAN?
What is matter?

- Article discussion

Different Interpretations

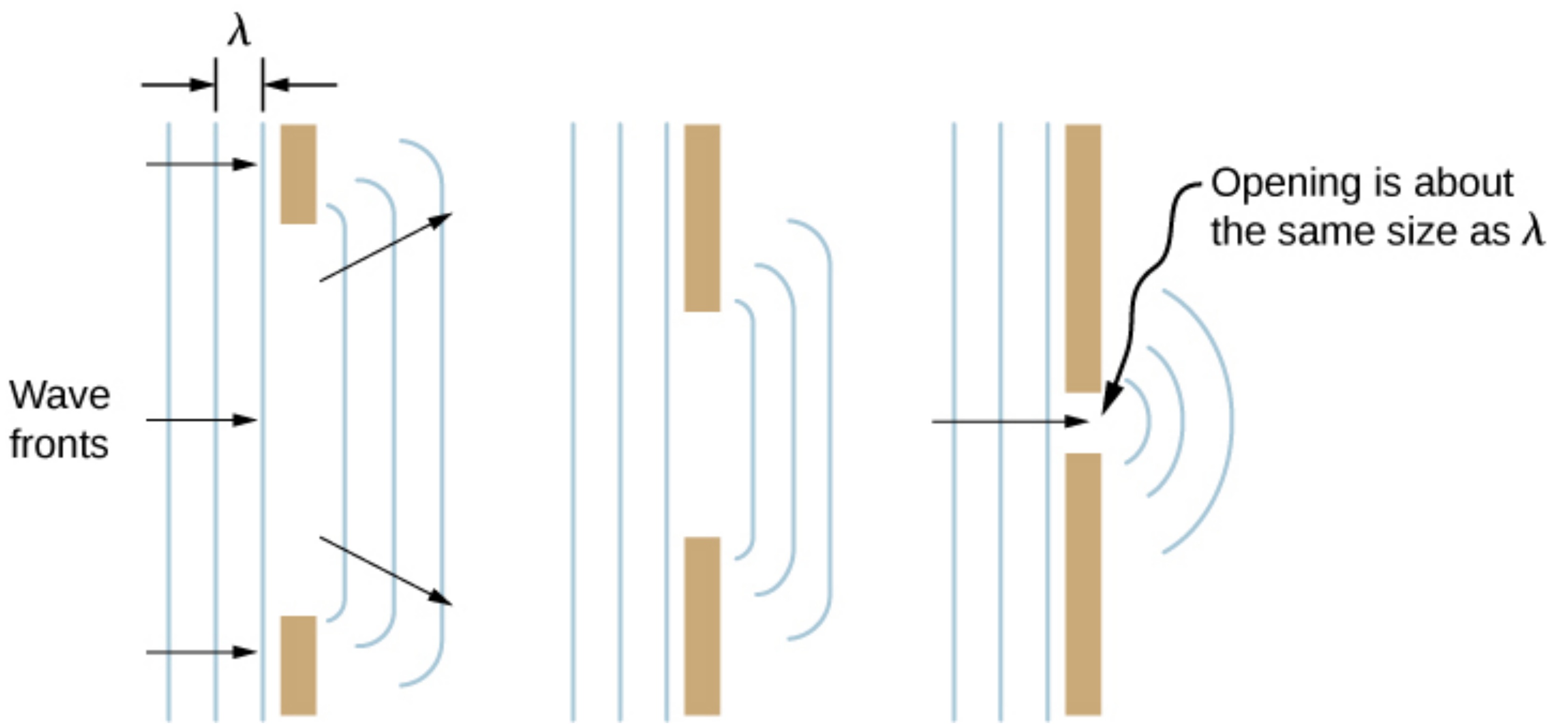
- Copenhagen Interpretation – the act of measurement affects the system. QM only predicts probabilities of measurements.
- Agnostic interpretation - "Don't know, don't care." – "Shut up and calculate."
- Many-World Interpretation – all possible and future histories are real.

Handout B: Wave Properties of Matter

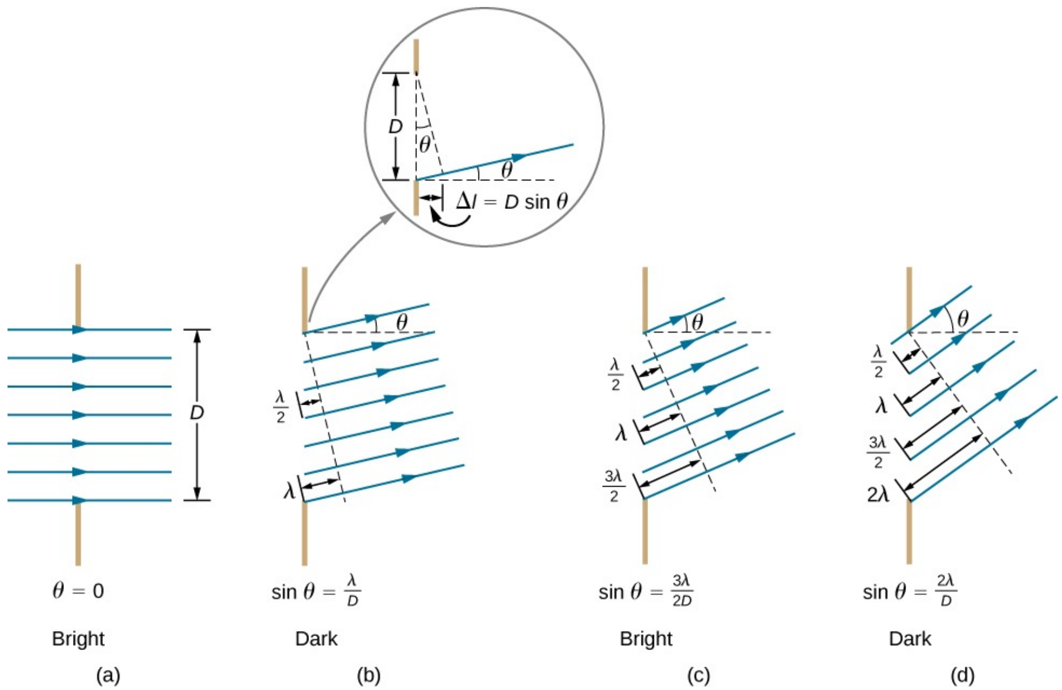
- Cover Davisson and Germer if time.
- Stopped here.

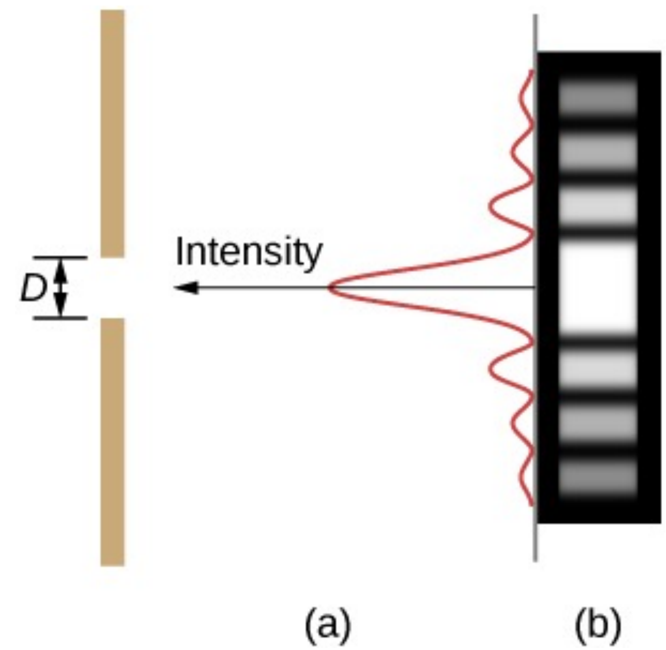
Diffraction

Recall Huygen's Principle



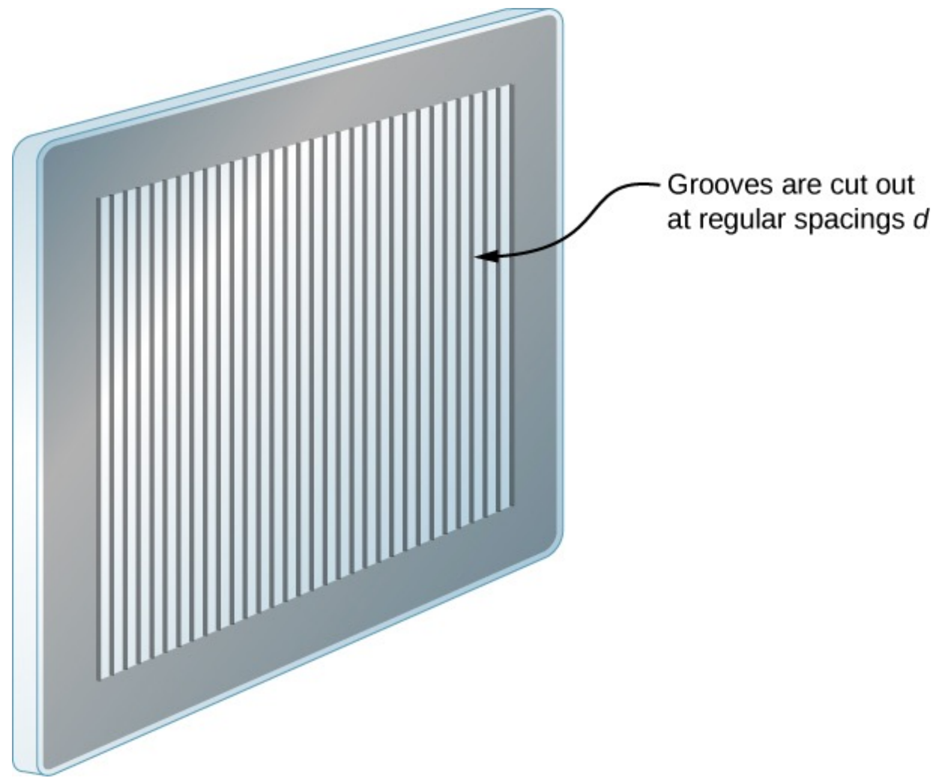
Diffraction: Single Slit, Wave "Interferes With Itself"



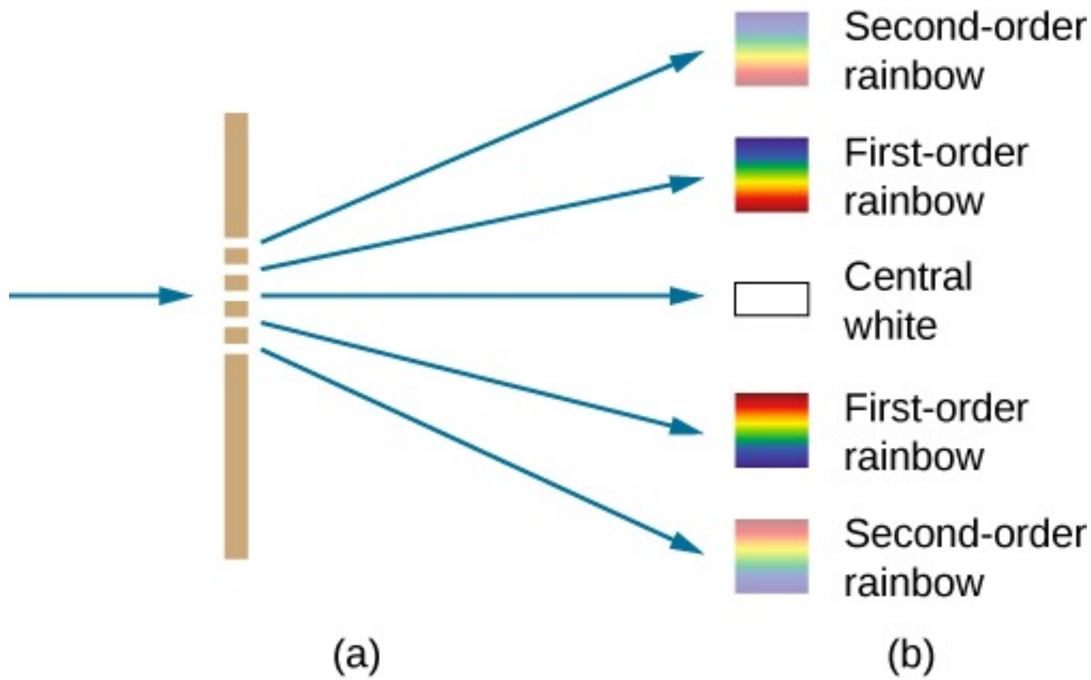


Diffraction Gratings

- What do you see?

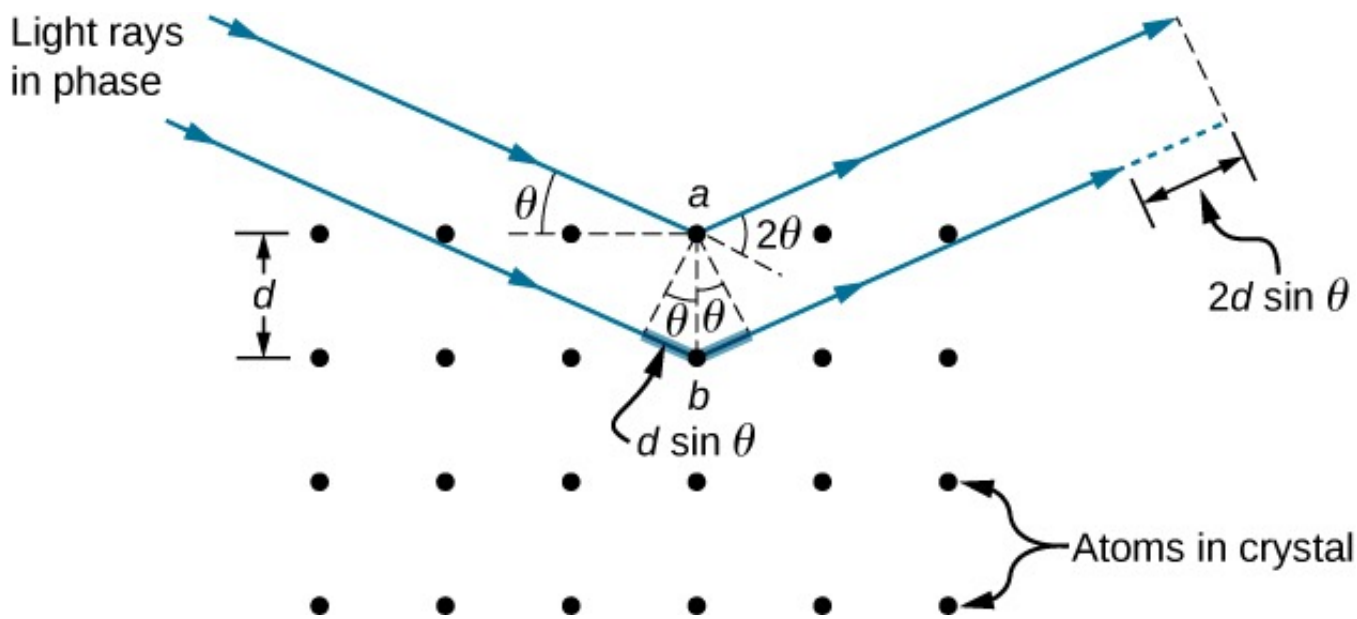


Diffraction Gratings



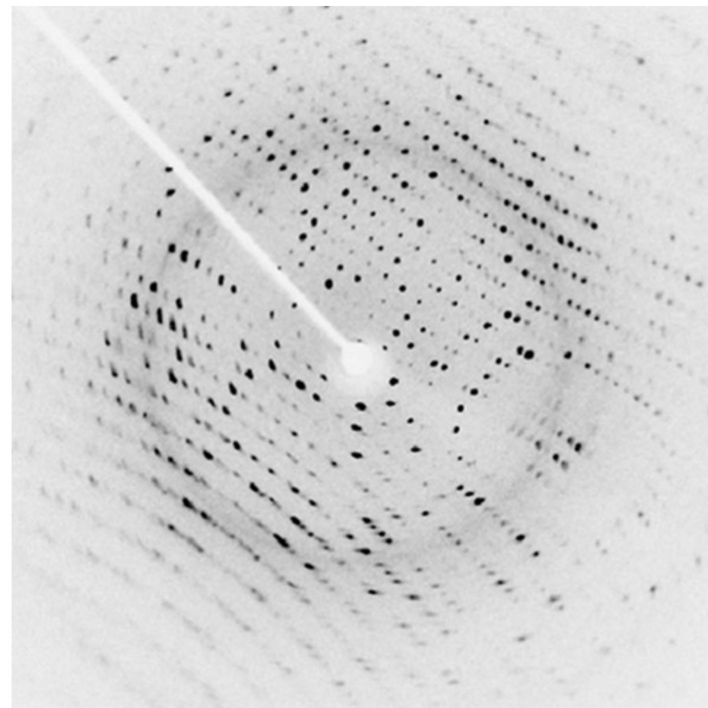
X-Ray Diffraction

- A regular crystaline structure of some materials can be used to produce diffraction of X-rays.



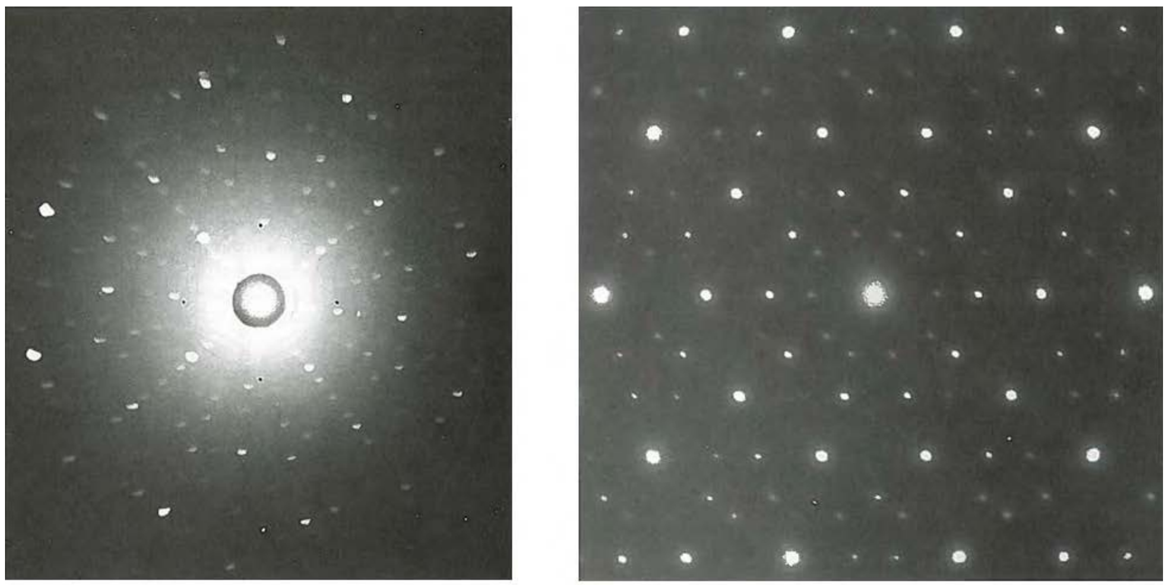
X-Ray Diffraction

- Right: X-ray diffraction from crystal of a protein.
- Proved X-rays were energetic EM waves in 1912 (Nobel Prize 1914)
- Nobel Prize 1915, Bragg & Bragg, X-ray analysis.
- Rosalind Franklin used X-ray diffraction to determine double-helix structure of DNA 1953. (Nobel Prize in Physiology or Medicine 1962 to Watson, Crick and Wilkins – not Franklin!)



Okay, so?

- The SAME BEHAVIOR was observed with electrons (accidentally) by Davisson and Germer.



(a) (b)

Figure 6.22 Diffraction patterns obtained in scattering on a crystalline solid: (a) with X-rays, and (b) with electrons. The observed pattern reflects the symmetry of the crystalline structure of the sample.