Phys 301 Class 24
Wavepackets, Heisenburg Uncertainty

Superposition

- If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to a wave equation, then so is:

$$\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t)$$

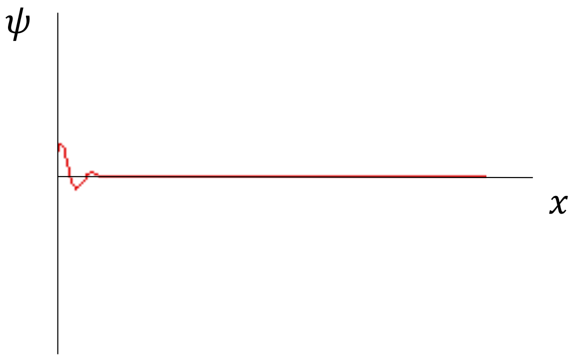
- Superposition (linear combination) of two waves.

Fourier Series

- A periodic function can be represented by the sum of simple sine and/or cosine functions.
- We’ve seen some examples of adding traveling waves to make “new” waves:
  - Standing Waves
  - Beats
- This is true for any periodic (repeating) function.

Wave Packets

- Our new representation of a “particle” in “free space”.
- Localization to be particle-like
- Oscillation to be wave-like

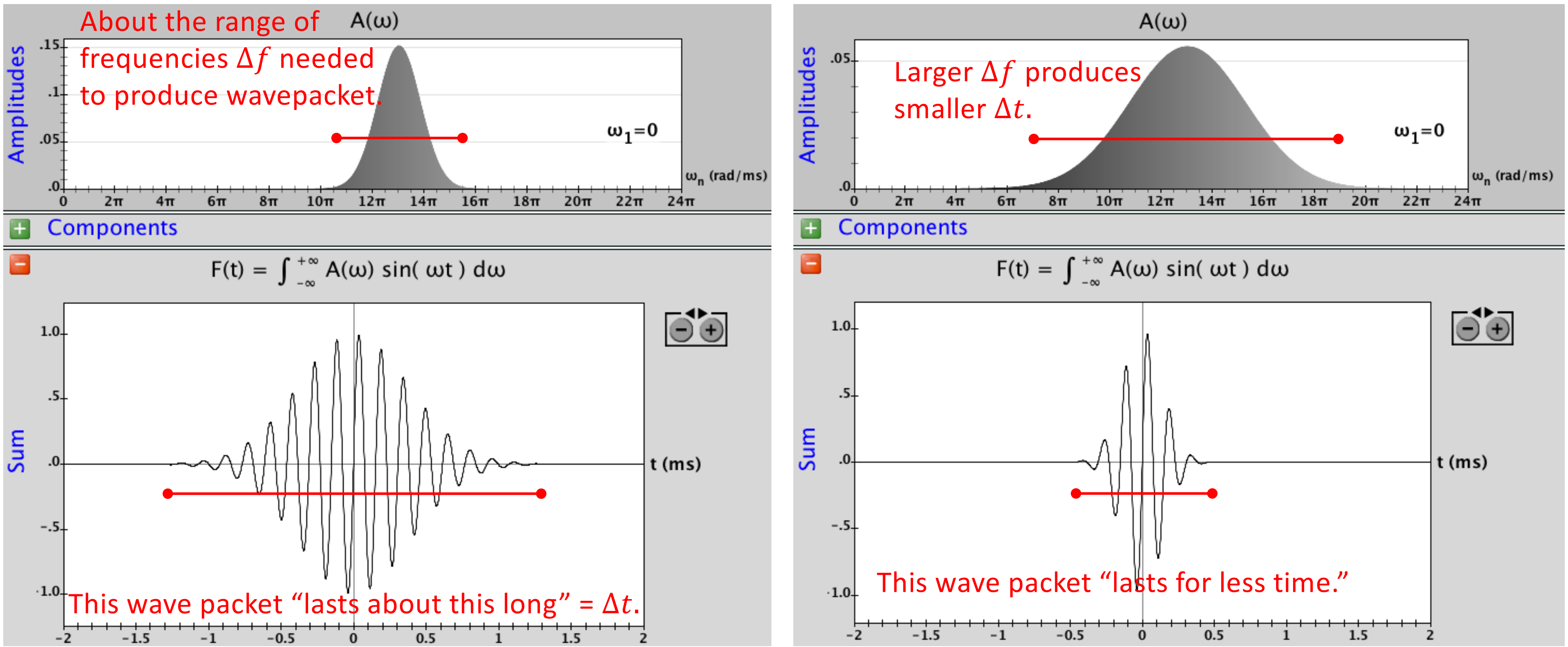


Time dimension is animated.
Animation is looped.

How To Make a Wave Packet?

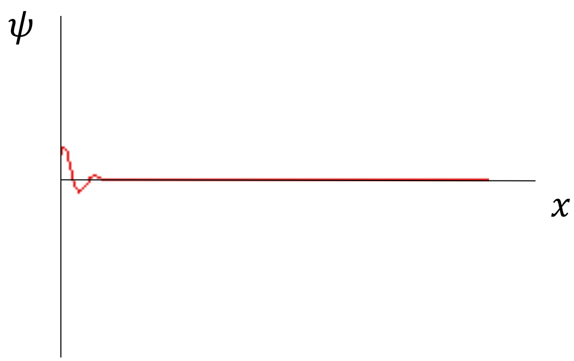
- Handout Parts I and II

When exactly does a wave packet "arrive" at a detector? (*e.g.*, an electron on our screen?) There is some uncertainty.



$$\Delta f \Delta t \geq 1$$

How do these uncertainties of wave properties apply to matter? Derivation on board.

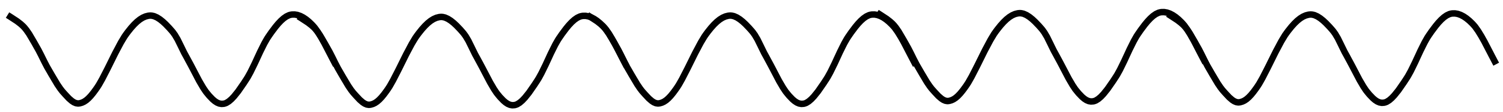


Time dimension is animated.
Animation is looped.

$$\Delta x \; \Delta p_x \geq h/2$$

Plane Waves vs. Wave Packets

$$\Psi(x,t) = A\ \exp[i(kx - \omega t)]$$



$$\Psi(x,t) = \sum_n A_n\ \exp[i(k_n x - \omega_n t)]$$

For which type of wave are the position (x) and momentum (p) most well-defined?

A) x most well-defined for plane wave, p most well-defined for wave packet.

B) p most well-defined for plane wave, x most well-defined for wave packet.

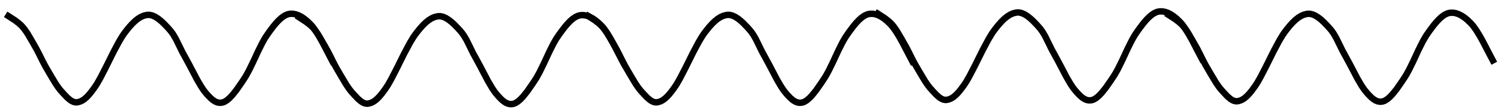
C) p most well-defined for plane wave, x equally well-defined for both.

D) x most well-defined for wave packet, p equally well-defined for both.

E) p and x are equally well-defined for both.

Uncertainty Principle

A Wave Interpretation:



Δx

small Δp – only one wavelength

Δx

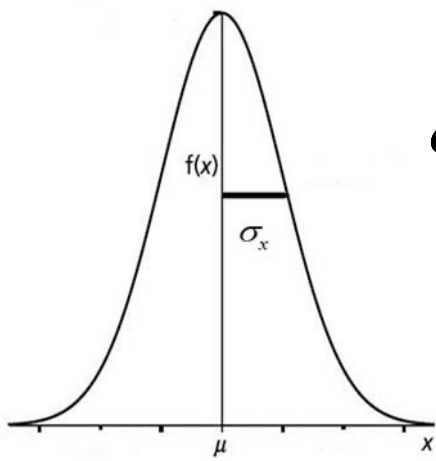
medium Δp – wave packet made of several waves

Δx

large Δp – wave packet made of lots of waves

Uncertainty Principle

A <u>Statistical</u> Interpretation:



$$\sigma_x = \frac{1}{N}\sum_{i=1}^{N}(\mu - x_i)^2$$

$$\sigma_x \cdot \sigma_p \geq \frac{\hbar}{2}$$

- Measurements are performed on an ensemble of identically prepared systems.
- Distributions of position and momentum values are obtained.
- Uncertainties in position and momentum are defined in terms of the standard deviation.

What about when we "observe" a particle?

- We more precisely determine its location.
- The *act* of observation *localizes* the electron.
  - We *change* the wavefunction.

The Implications

- Our knowledge about a particle's position and momentum is *inherently* uncertain.
- Independent of "experimental uncertainty."
- The more precisely we know a particle's position, the less precisely we know its momentum, and vice versa.

Matter Waves (Summary)

- Electrons and other particles have wave properties
  (interference)
- When not being observed, electrons are spread out in space
  (delocalized waves)
- When being observed, electrons are found in one place
  (localized particles)
- Particles are described by wave functions: $\Psi(x,t)$
  (probabilistic, not deterministic)
- Physically, what we measure is $\rho(x,t) = |\Psi(x,t)|^2$
  (probability density for finding a particle in a particular place at a particular time)
- Simultaneous measurements of x & p are constrained by the
  Uncertainty Principle: $\Delta x \Delta p_x \geq h/2$