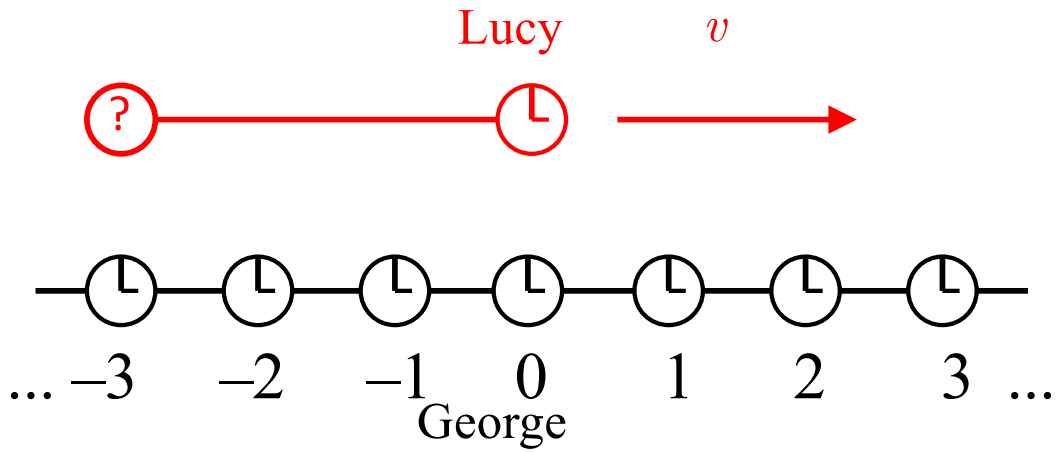
Phys 301 Modern Physics

Class 5: Spacetime Diagrams



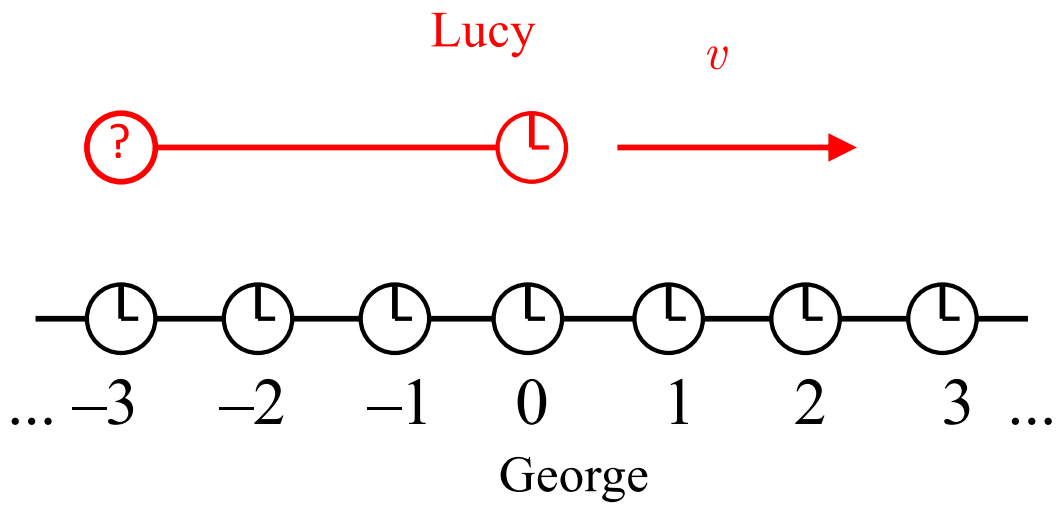
$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

George has a set of synchronized clocks in reference frame S, as shown. Lucy is moving to the right past George and has (naturally) her own set of synchronized clocks. Lucy passes George at the event (0 m, 0 s) in both frames. An observer in George's frame checks the clock marked '?'. Compared to George's clocks, this one reads

A) a slightly earlier time

B) a slightly later time

C) same time



$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

The event has coordinates (x = –3m, t = 0s) for George.

In Lucy's frame, where the ? clock is, the time t' is

$$t' = \gamma\left(0 - \frac{v}{c^2}(-3\text{m})\right) = (+3\text{m})\frac{\gamma v}{c^2}$$, a positive quantity.

B) a slightly later time

Lorentz Velocity Transformations

Frame S′ moves relative to S.

An object is moving through both frames at velocity u and u', respectively.

$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$u' = dx'/dt'$

Use expressions for dx' and dt', take derivative to find u' in terms of u, v, c.

Lorentz Velocity Transformations

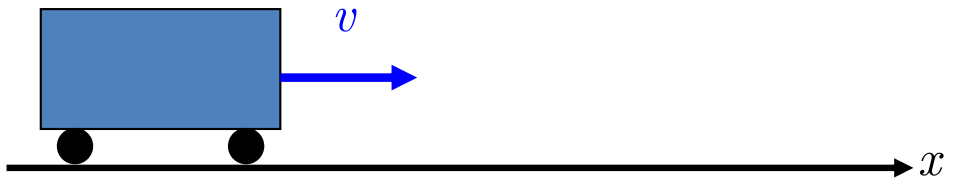
$$u' = \frac{u - v}{1 - uv/c^2}$$

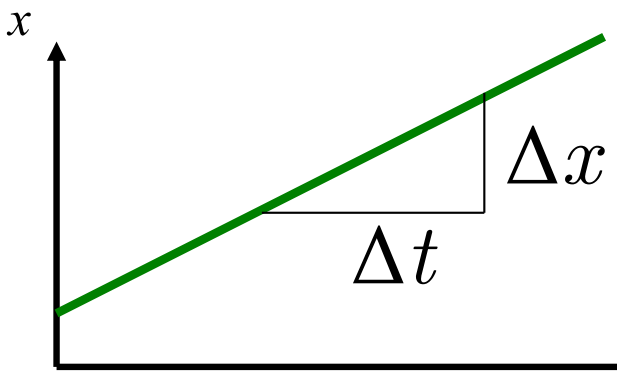
$$u = \frac{u' + v}{1 + u'v/c^2}$$

Must distinguish v, u, and u'!

Spacetime Diagrams (1D in space)

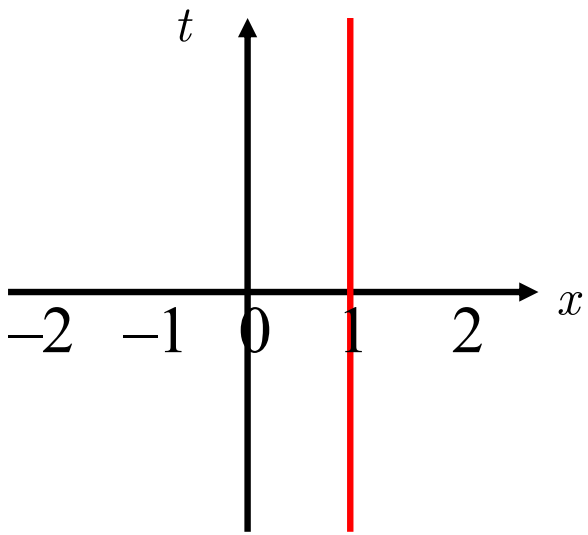
In Phys 211:



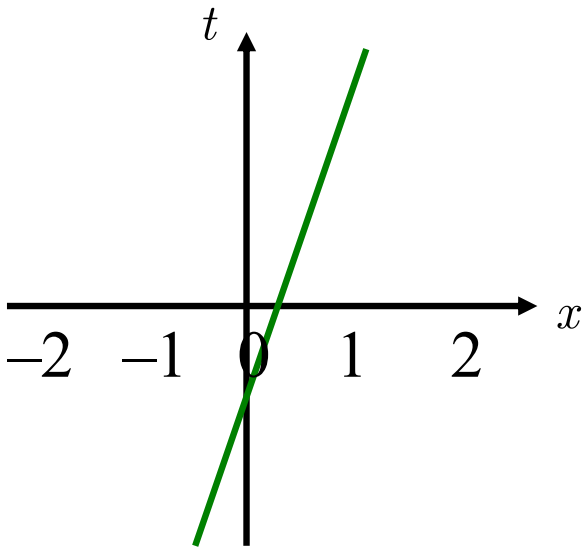


$$v = \Delta x / \Delta t$$

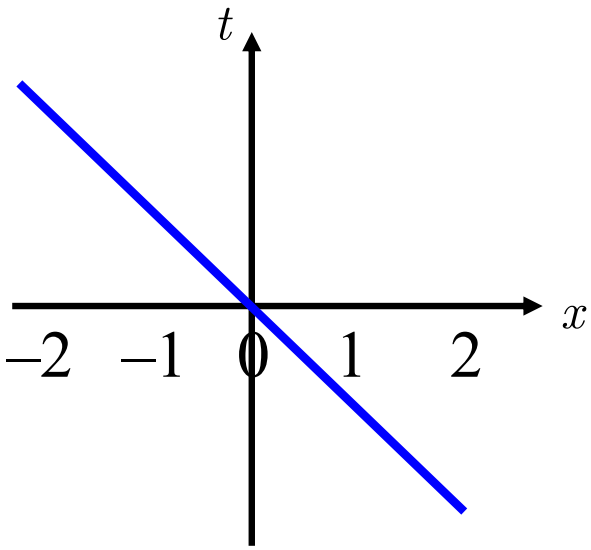
Representing Objects in Spacetime Diagrams (1D in space)



Object at rest at $x = 1$.



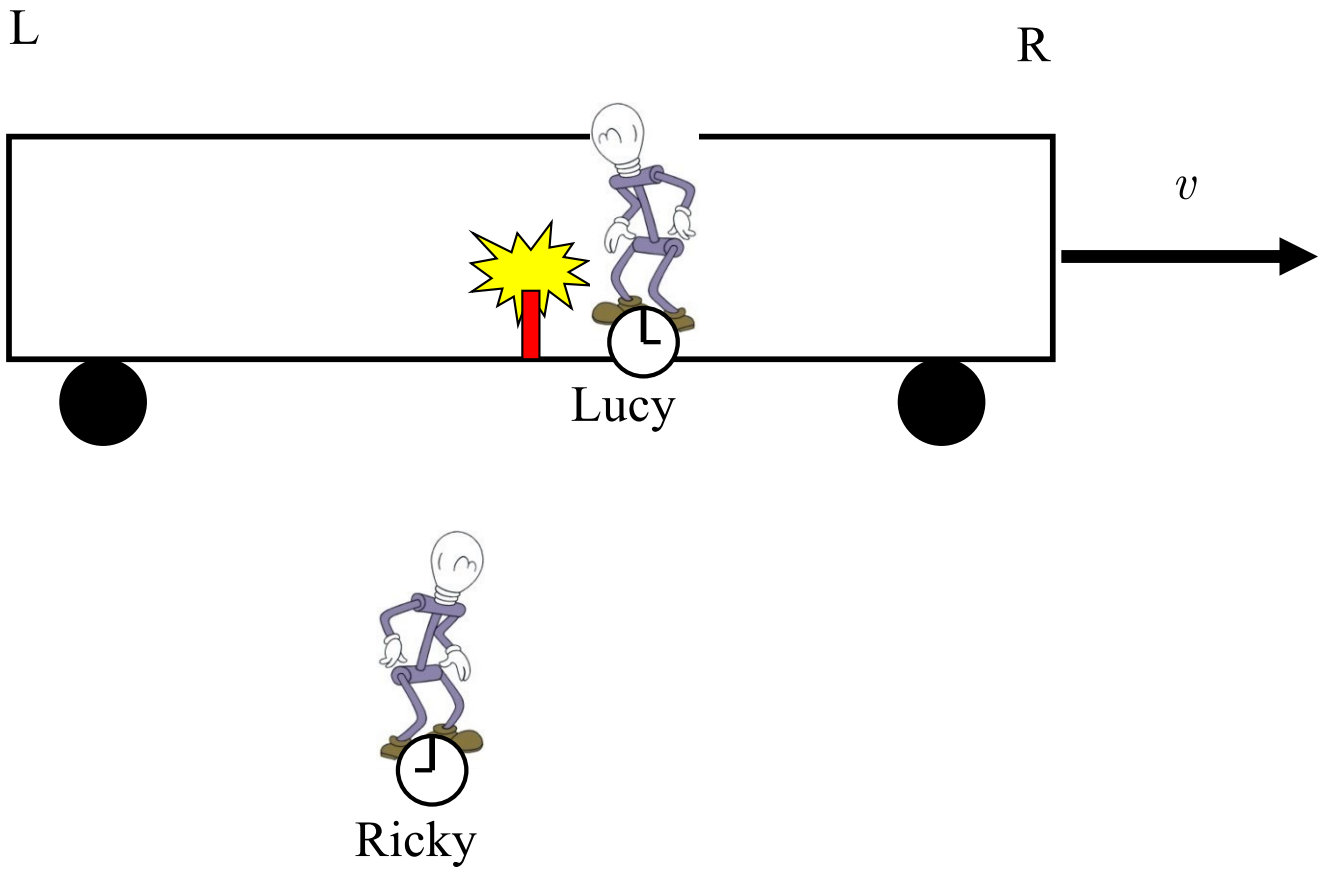
Object moving with $0 < v < c$. $x > 0$ at time $t = 0$.



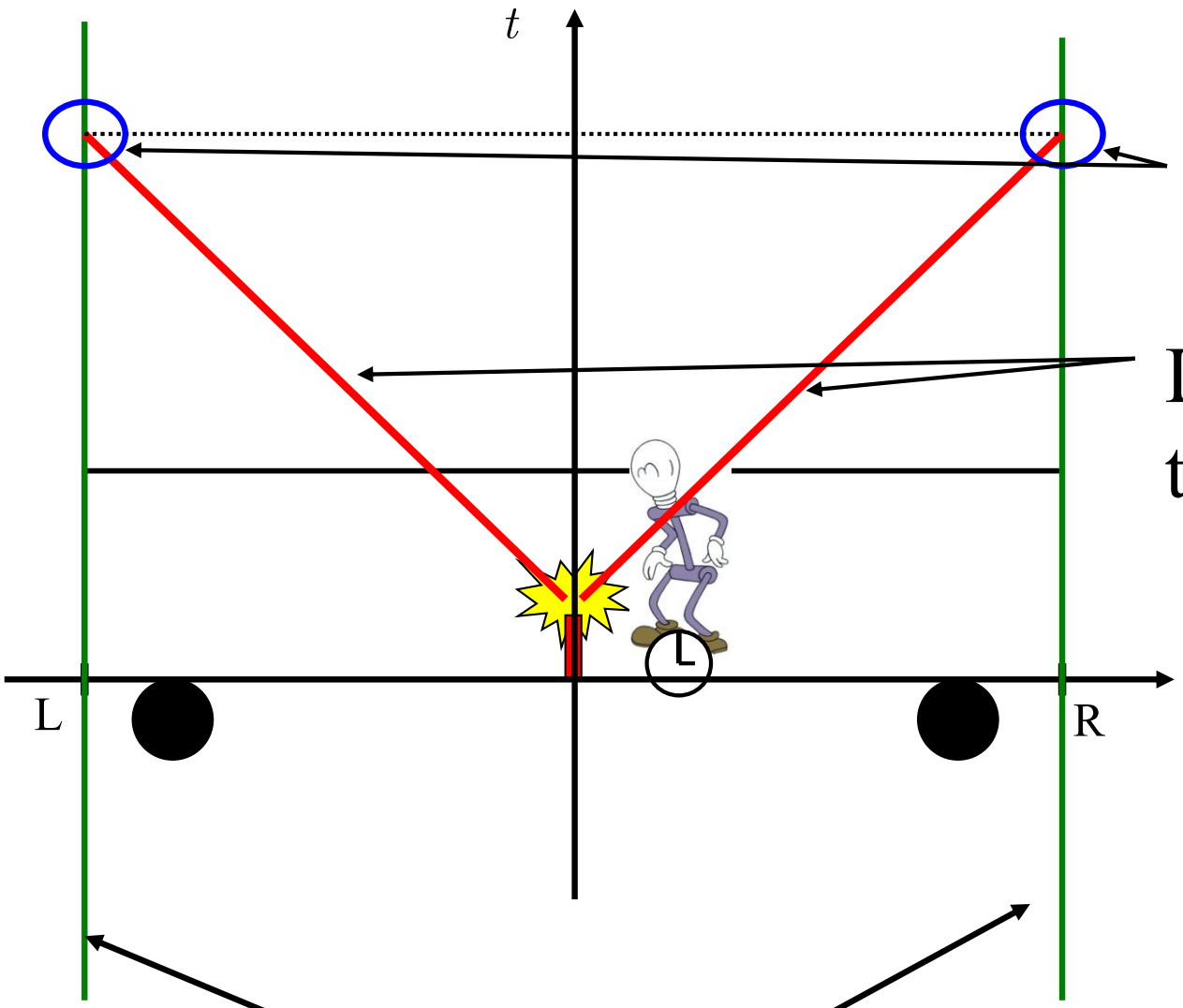
Object moving with $v = -c$. $x = 0$ at time $t = 0$

"Worldlines"

Recall: Lucy plays with a fire cracker in the train. Ricky watches the scene from the track.



Example: Lucy in the train



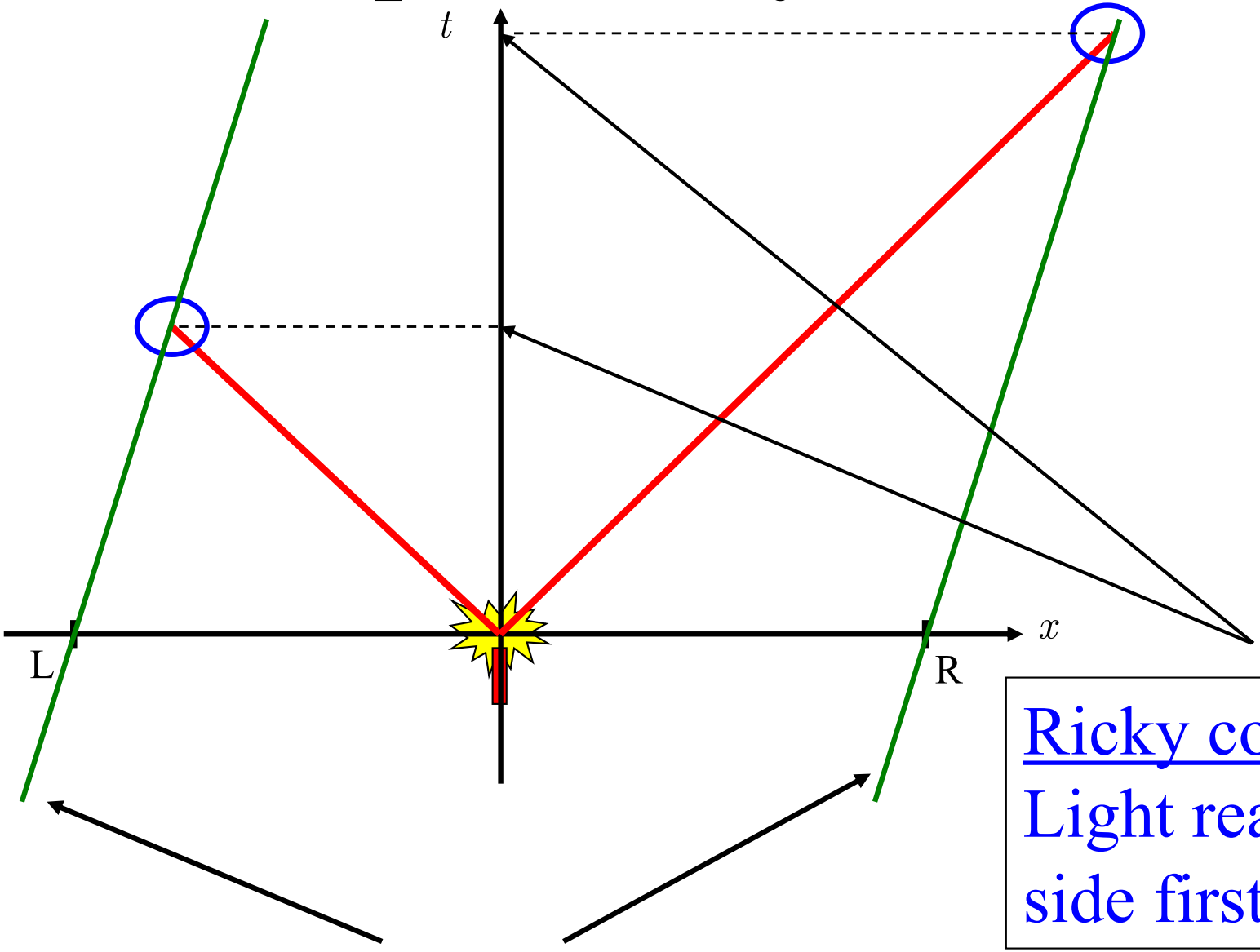
Light reaches both walls at the same time.

Light travels to both walls

In Lucy's frame: Walls are at rest

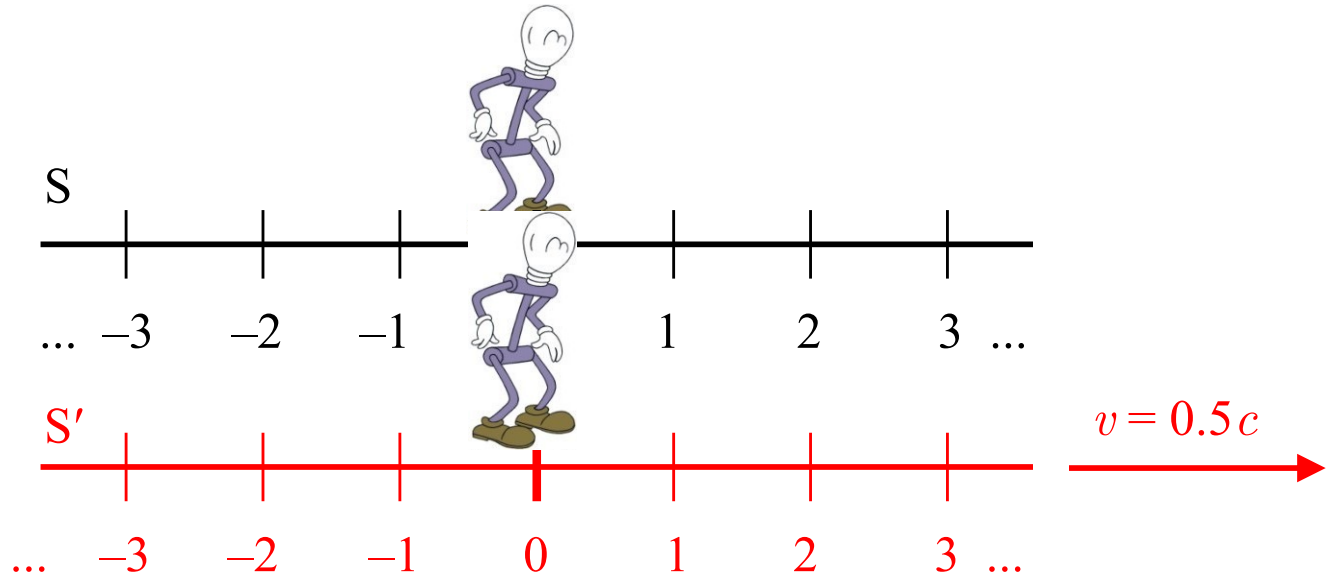
Lucy concludes: Light reaches both sides at the same time.

Example: Ricky on the tracks

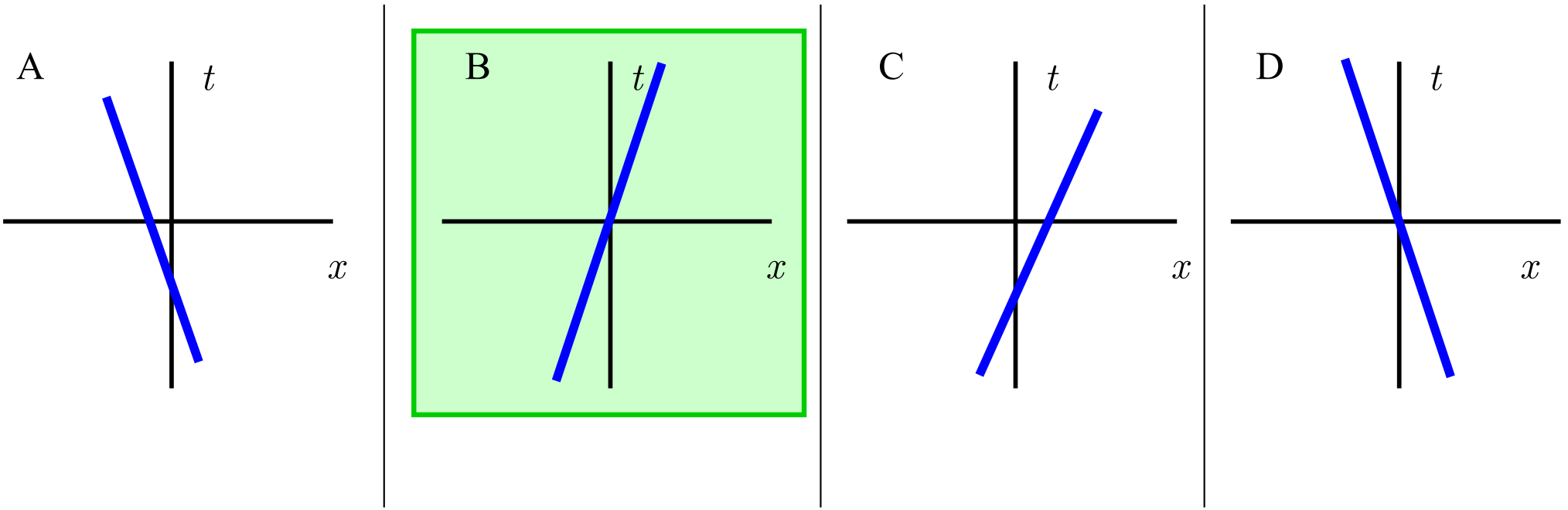


Ricky concludes:
Light reaches left side first.

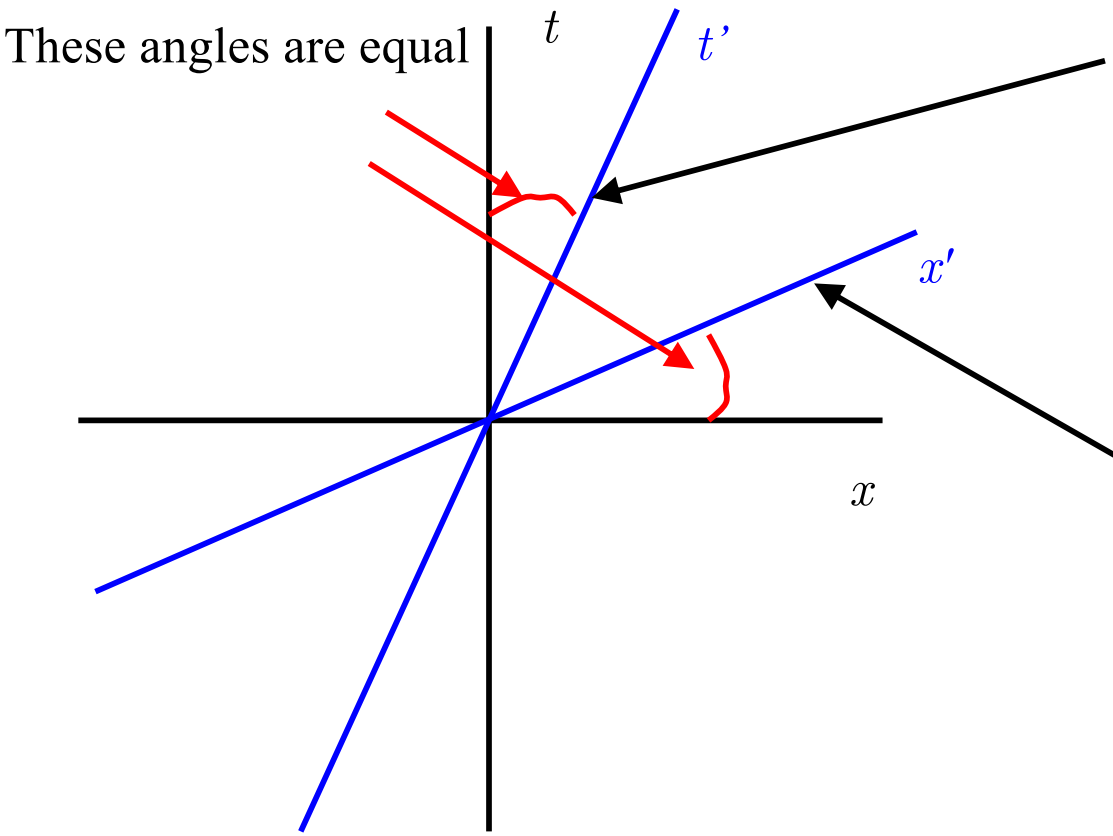
In Ricky's frame: Walls are in motion



Frame S′ is moving to the right at $v = 0.5\,c$. The origins of S and S′ coincide at $t = t' = 0$. Which shows the world line of the origin of S′ as viewed in S?



Frame S' as viewed from S

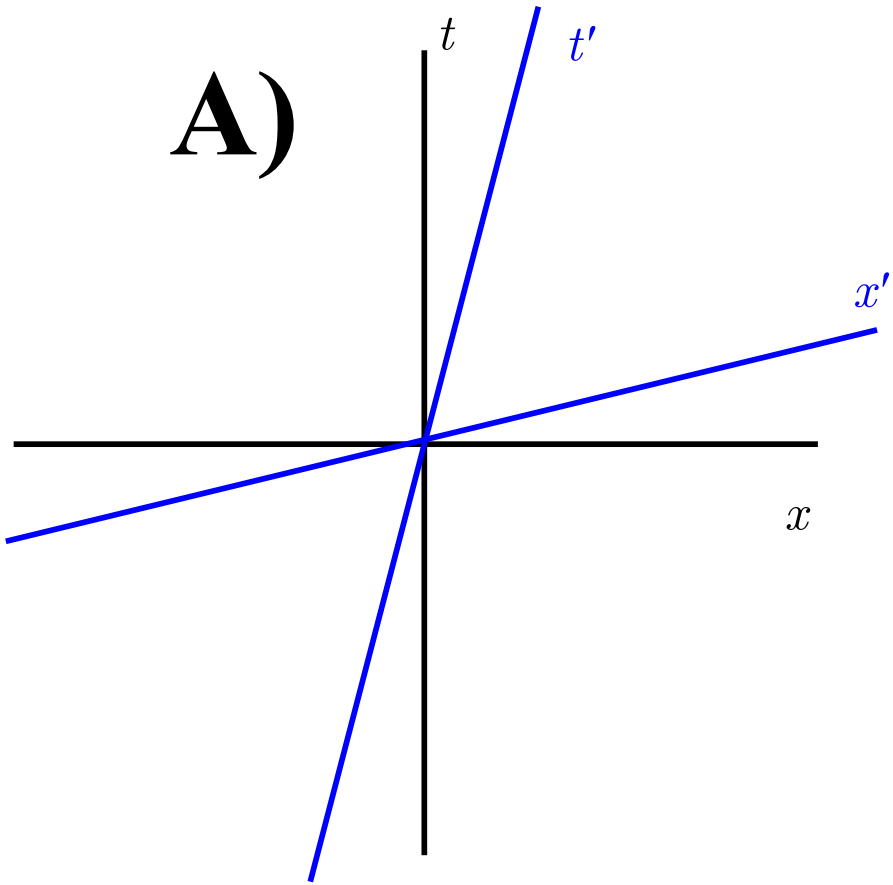


This is the time axis of the frame S′
Slope = $1/\beta$

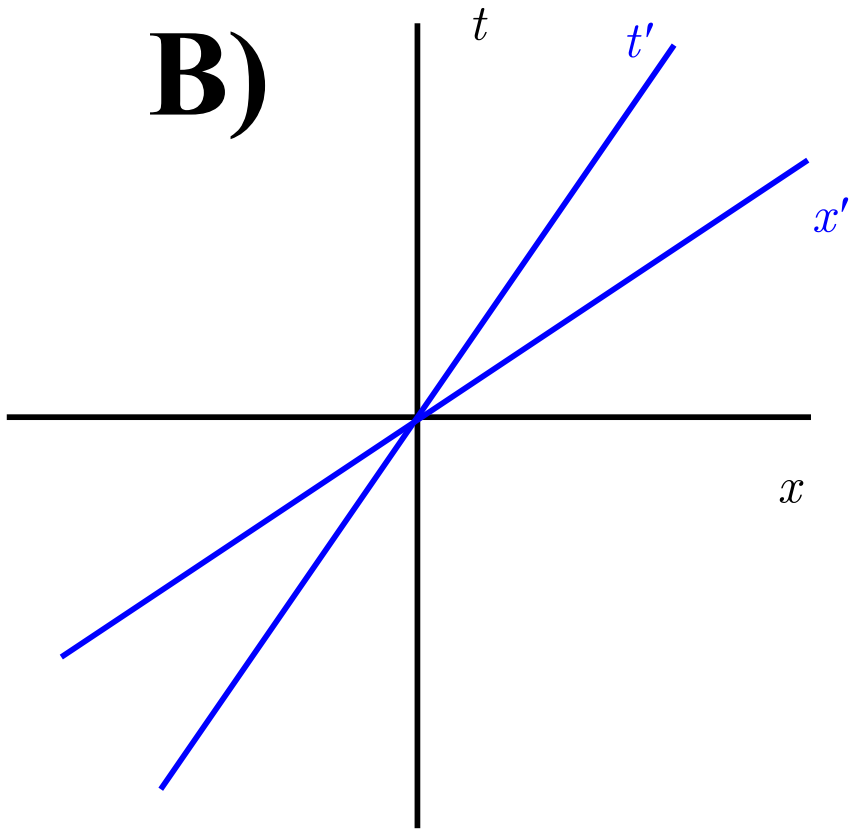
This is the space axis of the frame S′

Which diagram shows the S′ frame moving the fastest relative to S?

A)

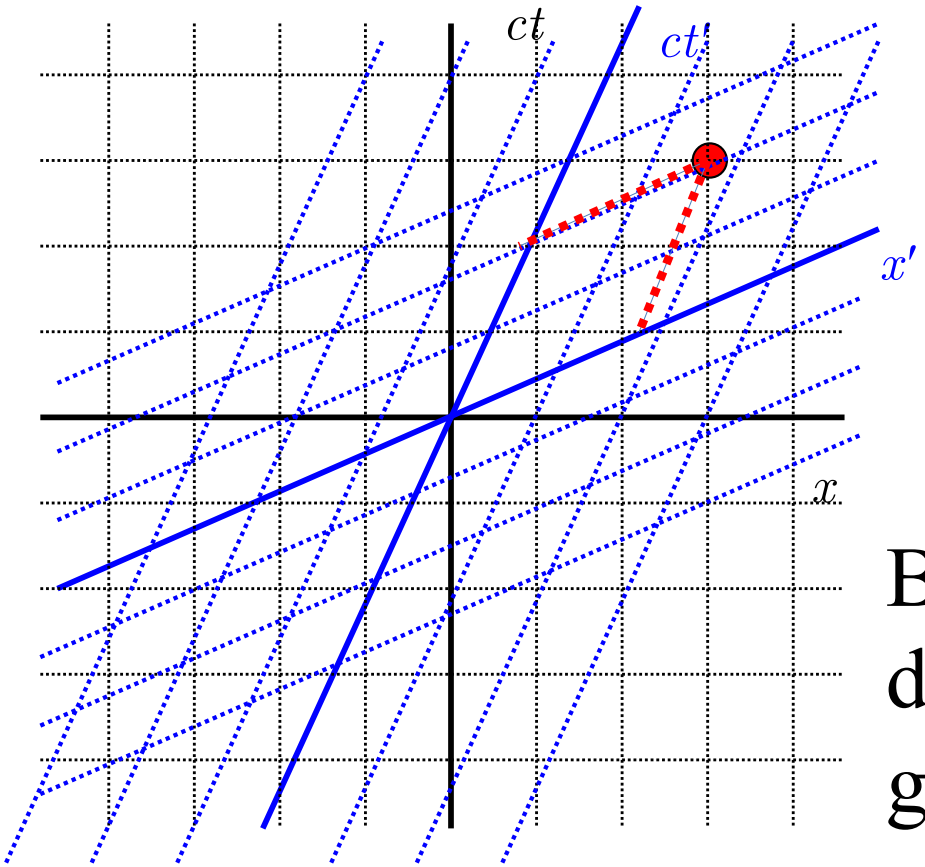


B)



C) Need more info.

Frame S′ as viewed from S



In S: (3 m, 3 m)

In S′: (1.8 m, 2 m)

Both frames are adequate for describing events – but will give different spacetime coordinates for these events, in general.

In-Class Activities

- Barn in Pole Paradox
- Break?
- More Paradoxes