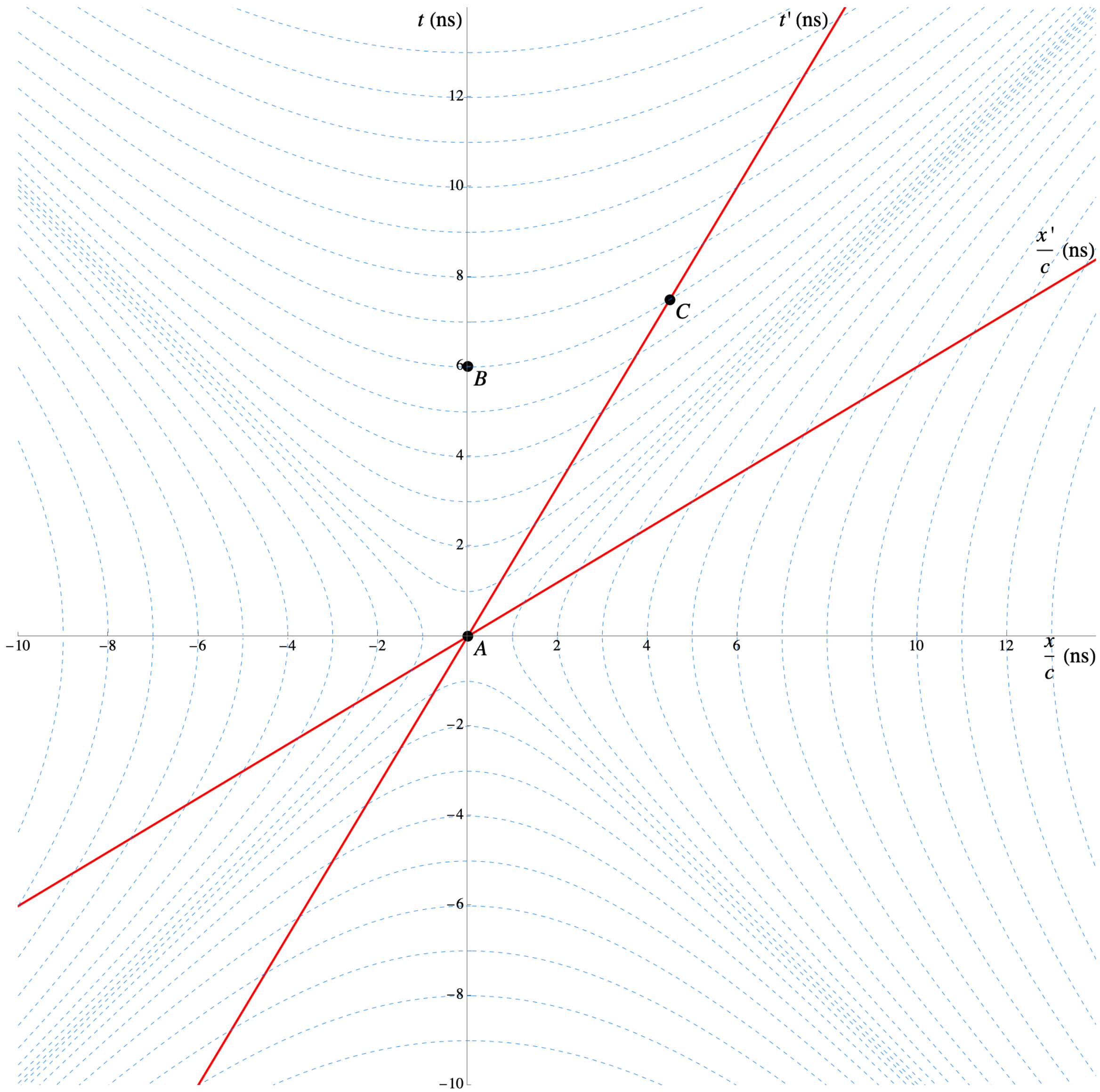
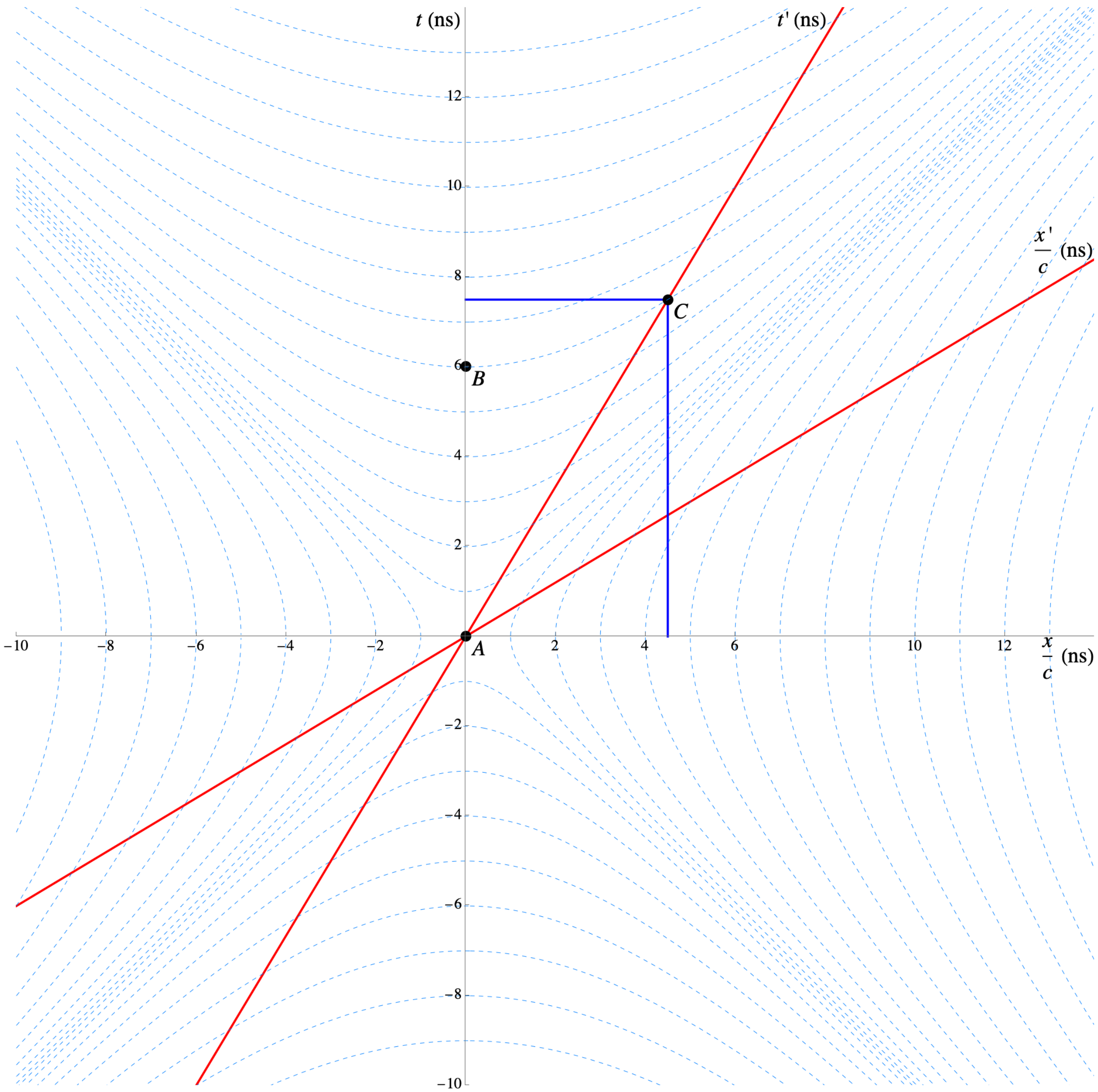
Finish Up Paradoxes

- ~ 15 min. Work on “How can both observers see the other’s clock running slow?”
- Follow the hints.


t (ns)
t' (ns)
x'/c (ns)
x/c (ns)
A
B
C
−10
−8
−6
−4
−2
2
4
6
8
10
12

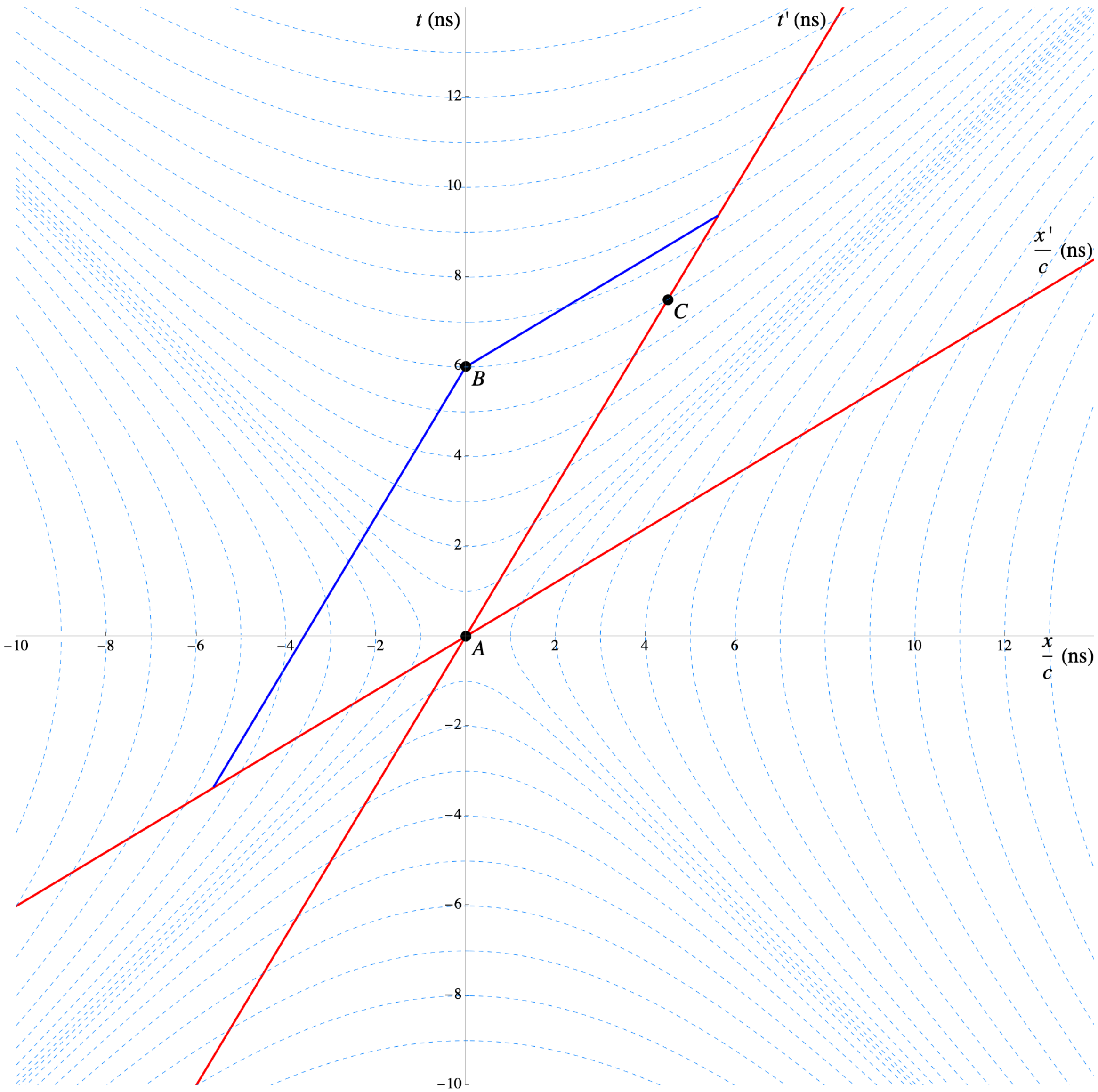


Check with Lorentz transformations:

Event C in S frame:

$$x = \gamma(x' + vt') = 1.25\left(0 + \frac{3}{5}(6\text{ s})\right) = 4.5\text{ s}$$

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right) = 1.25\left(6\text{ s} + \frac{3}{5}(0)\right) = 7.5\text{ s}$$



Check with Lorentz transformations:

Event B in S′ frame :

$$x' = \gamma(x - vt) = 1.25\left(0 - \frac{3}{5}(6\text{ s})\right) = -4.5\text{ s}$$

$$t' = \gamma\left(t - \frac{v}{c^2}x'\right) = 1.25\left(6\text{ s} - \frac{3}{5}(0)\right) = 7.5\text{ s}$$

Warm-Up Problem: Lorentz Transformations On Handout

The clock travels from A to B with speed v.

Assume A is at position $x = 0$, then B is at position $x = vt$, $t = (t_1 - t_0)$

Use this to substitute x in the Lorentz transformation:

$$t' = \gamma \left(t - \frac{v^2 t}{c^2} \right)$$

$$= \gamma t \left(1 - \frac{v^2}{c^2} \right) = \frac{t}{\gamma}$$

The moving clock shows the proper time interval!!

$$\Delta\tau = \frac{\Delta t}{\gamma}$$

→ **We get exactly the expression of the time dilation!**

Phys 301 Modern Physics

Class 6: Momentum and Energy

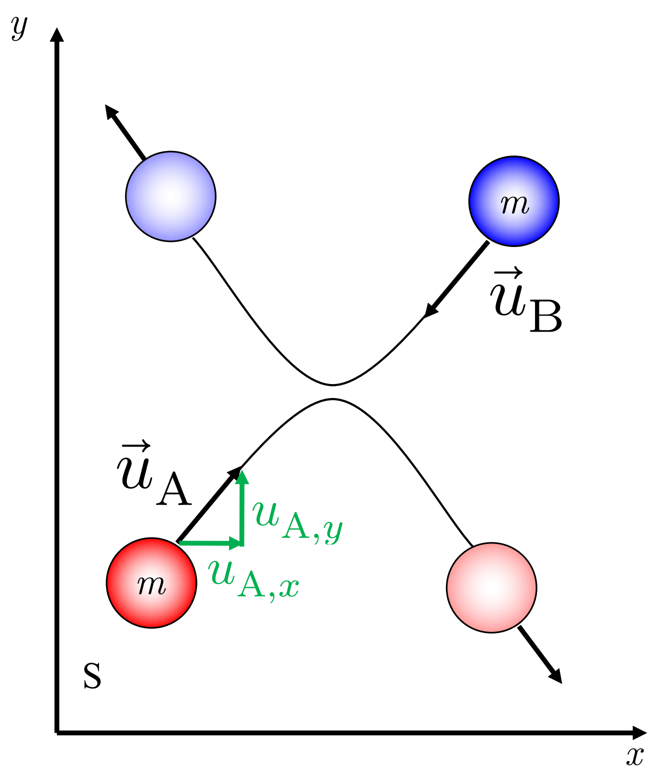
Conservation of Momentum: Classical

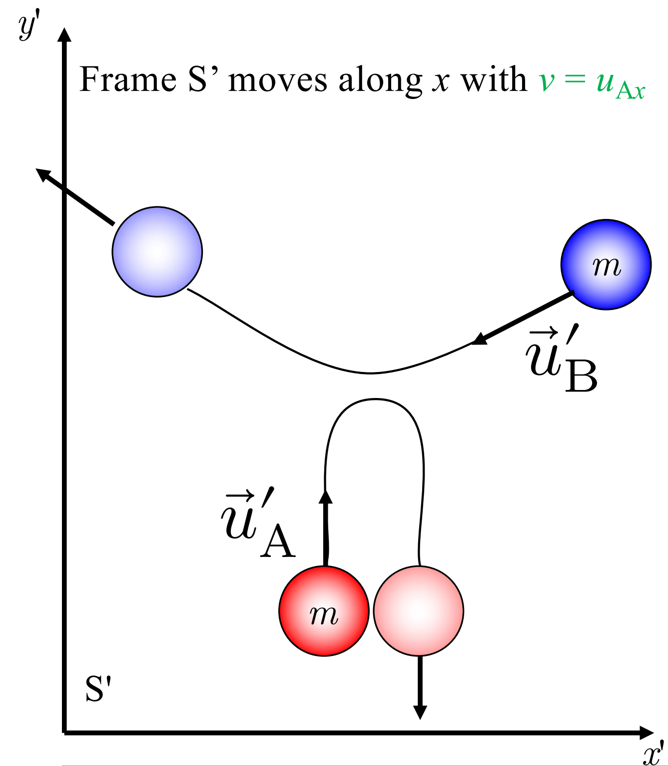
$$\vec{p} = m\vec{u}$$

In absence of external forces the total momentum is conserved (Law of conservation of momentum):

$$\sum_{i=1}^{n} \vec{p}_i = \text{const.}$$

Conservation of Momentum



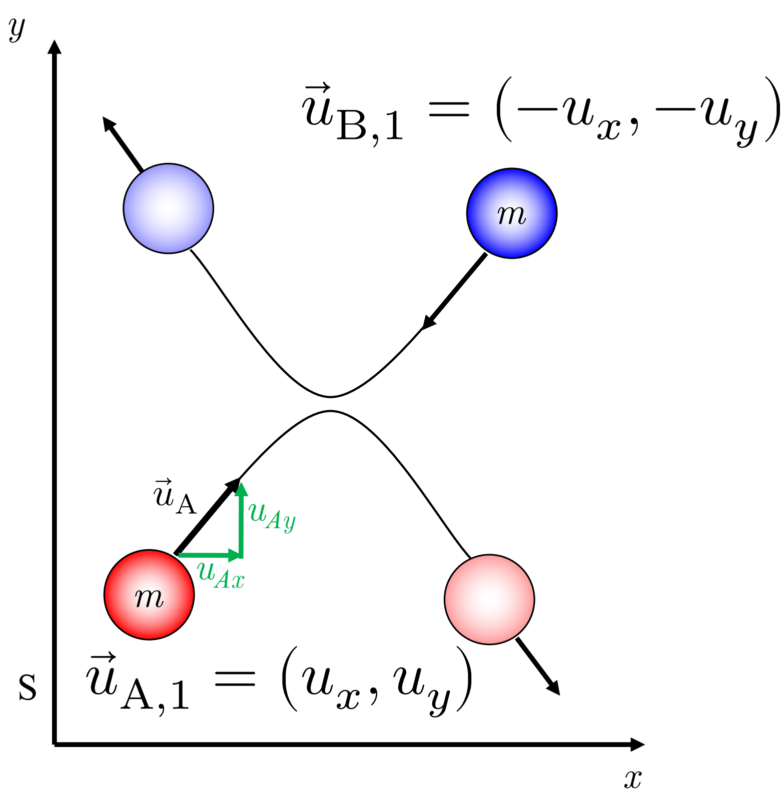


If $\vec{u}_B = -\vec{u}_A$ we find:
$p_{tot,1} = 0$
$p_{tot,2} = 0$

System S' is moving to the right with the velocity $v = u_{Ax}$.

Classical Momentum

Write complete vector expressions using m, u_x, u_y.

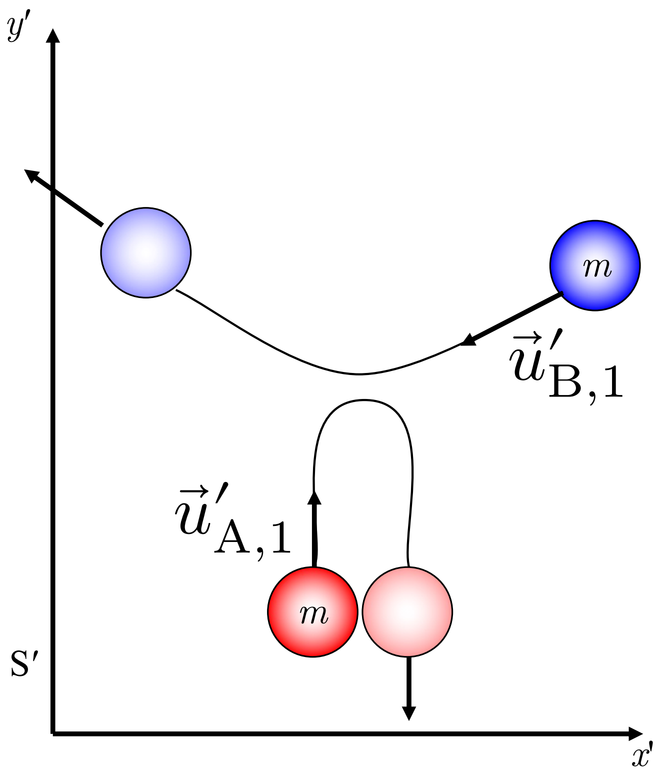


$$\vec{p}_{\mathrm{A},1} = m(+u_x\hat{x} + u_y\hat{y})$$
$$\vec{p}_{\mathrm{B},1} = m(-u_x\hat{x} - u_y\hat{y})$$
$$\vec{p}_{\mathrm{tot},1} = m(\ \ 0\hat{x}\ +\ 0\hat{y})$$

$$\vec{p}_{\mathrm{A},2} = m(+u_x\hat{x} - u_y\hat{y})$$
$$\vec{p}_{\mathrm{B},2} = m(-u_x\hat{x} + u_y\hat{y})$$
$$\vec{p}_{\mathrm{tot},2} = m(\ \ 0\hat{x}\ +\ 0\hat{y})$$

$$\boxed{\rightarrow \vec{p}_{\mathrm{tot},1} = \vec{p}_{\mathrm{tot},2}}$$

Using Galilean Transformations



Write expressions using the same u_x, u_y as before.

$$\vec{p}'_{A,1} = m(\quad 0\hat{x} \quad + u_y\hat{y})$$
$$\vec{p}'_{B,1} = m(-2u_x\hat{x} - u_y\hat{y})$$
$$\vec{p}'_{tot,1} = m(-2u_x\hat{x} + \; 0\hat{y})$$

$$\vec{p}'_{A,2} = m(\quad 0\hat{x} \quad - u_y\hat{y})$$
$$\vec{p}'_{B,2} = m(-2u_x\hat{x} + u_y\hat{y})$$
$$\vec{p}'_{tot,2} = m(-2u_x\hat{x} + \; 0\hat{y})$$

$$\boxed{\rightarrow \vec{p}'_{tot,1} = \vec{p}'_{tot,2}}$$

Velocity Transformation (3D)

<u>Classical:</u>

$$u'_x = u_x - v$$

$$u'_y = u_y$$

$$u'_z = u_z$$

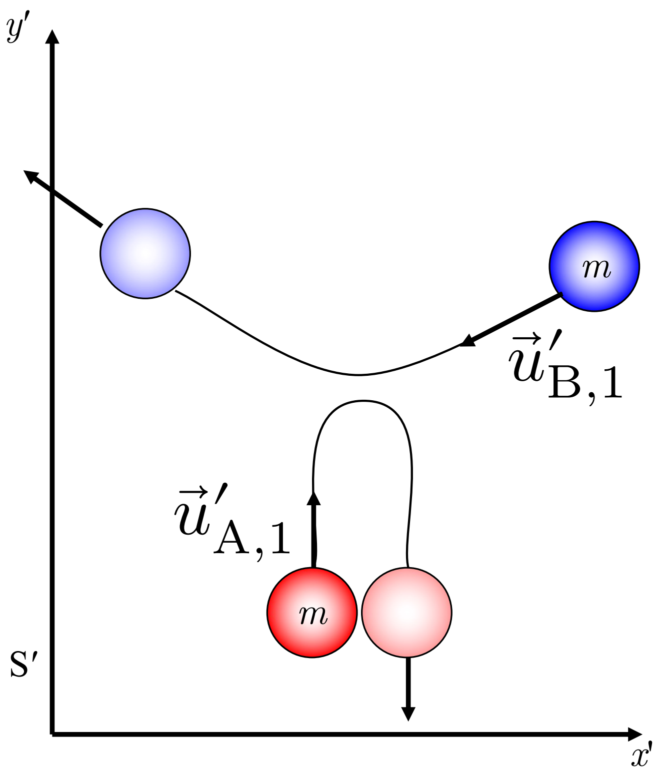
<u>Relativistic:</u>

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)}$$

$$u'_z = \frac{u_z}{\gamma(1 - u_x v/c^2)}$$

Lorentz Transformation



Use: $$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)}$$

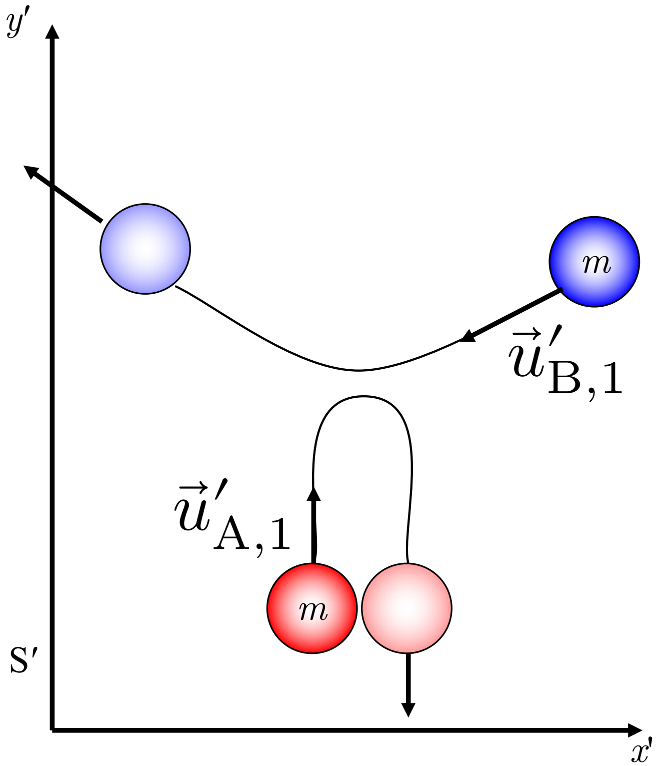
Does ***initial total momentum*** equal ***final total momentum?***

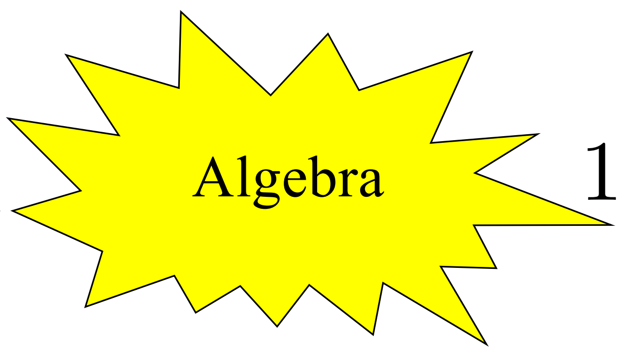
$$\vec{p}'_{\mathrm{A},1} = m\left(0\hat{x} + \frac{+u_y}{\gamma(1-u_x^2/c^2)}\hat{y} \right)$$

$$\vec{p}'_{\mathrm{B},1} = m\left(\frac{-2u_x}{\gamma(1+u_x^2/c^2)}\hat{x} + \frac{-u_y}{\gamma(1+u_x^2/c^2)}\hat{y} \right)$$

$$\vec{p}'_{\mathrm{A},2} = m\left(0\hat{x} + \frac{-u_y}{\gamma(1-u_x^2/c^2)}\hat{y} \right)$$

$$\vec{p}'_{\mathrm{B},2} = m\left(\frac{-2u_x}{\gamma(1+u_x^2/c^2)}\hat{x} + \frac{+u_y}{\gamma(1+u_x^2/c^2)}\hat{y} \right)$$





$$1 + \frac{u_x^2}{c^2} \neq 1 - \frac{u_x^2}{c^2}$$

$$p'_{x,1} = p'_{x,2}$$

$$p'_{y,1} \neq p'_{y,2}$$

Qualifications for New Definition of Momentum

1. At low velocities the new definition of $\vec{p}$ should match the classical definition of momentum.

2. The total momentum ($\Sigma\,\vec{p}_i$) of an isolated system of bodies is conserved in **all** inertial frames.

Relativistic Momentum

Classical definition: $\vec{p} = m\dfrac{d\vec{r}}{dt}$

- Measure the mass 'm' in its rest-frame ('*proper mass'* or '*rest mass'*).
- Agree on the same value for 'm' **in all frames**.
- Take derivative w.r.t. proper time.

Relativistic definition: $\vec{p} = m\dfrac{d\vec{r}}{dt_{\text{proper}}} = m\dfrac{d\vec{r}}{d\tau}$

Relativistic Momentum

$$dt = \gamma_p d\tau$$

$$\gamma_p = \frac{1}{\sqrt{1 - u_p^2/c^2}}$$

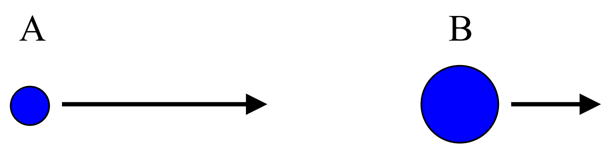
- Subscript "*p*" indicates "Lorentz factor of particle" – use speed of particle.

$$\vec{p} = \gamma_p m \frac{d\vec{r}}{dt} = \gamma_p m \vec{u}$$

By How Much does Relativistic Formula Change Momentum? Handout Part I

Relativistic Momentum

$$p = \gamma_p m u$$



Particle A has half the mass and twice the speed of particle B. If the particles' momenta are p_{A} and p_{B}, then

a) $p_{\mathrm{A}} > p_{\mathrm{B}}$
b) $p_{\mathrm{A}} = p_{\mathrm{B}}$
c) $p_{\mathrm{A}} < p_{\mathrm{B}}$

γ_p is bigger for the faster particle.

Relativistic Force: Handout Part II

Consequence of Lorentz factor:
nothing can be accelerated past speed of light.

$$u = \frac{Fct}{\sqrt{(Ft)^2 + (mc)^2}}$$