Phys 301 Class 09: Wave Velocity, Energy

Take out the handout from last class and open up the accompanying Excel file.

Wrap-Up Last Class (pg. 7)

- **Any function with the argument ($x \pm vt$) that is twice differentiable can represent a traveling wave.**
- (Q21,22) $kx - \omega t = k\left(x - \left(\frac{\omega}{k}\right)t\right) = k(x - vt)$
- Speed $|v|$ to the right.

Is ω/k really equal to v?

$$|v_x^{\text{wave}}| = v^{\text{wave}} = \frac{\lambda}{T} = \frac{\omega}{k}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad k = \frac{2\pi}{\lambda}$$

A function, $f(x,t)$, satisfies this PDE:

$$\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}$$

Invent two different functions $f(x,t)$ that solve this equation. Try to make one of them "boring" and the other "interesting" in some way.

A function, $f(x,t)$, satisfies the wave equation:

$$\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}$$

Which of the following functions work?

A) $f(x,t) = \sin\left(k(x - vt)\right)$

B) $f(x,t) = \exp\left(k(-x - vt)\right)$

C) $f(x,t) = A(x + vt)^3$

D) All of these

A "right moving" solution to the wave equation is:

$$f_R(z,t) = A\cos(kz - \omega t + \delta)$$

Which of these do you prefer for a "left moving" soln? (Assume k, ω, δ are positive quantities.)

A) $f_L(z,t) = A\cos(kz + \omega t + \delta)$

B) $f_L(z,t) = A\cos(kz + \omega t - \delta)$

C) $f_L(z,t) = A\cos(-kz - \omega t + \delta)$

D) $f_L(z,t) = A\cos(-kz - \omega t - \delta)$

E) more than one of these

Two different functions $f_a(x,t)$ and $f_b(x,t)$ are solutions of the wave equation:

$$\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}$$

Is $A\,f_a(x,t) + B\,f_b(x,t)$ also a solution of the wave equation?

A) Yes, always.
B) No, never.
C) Yes, sometimes, depending of f_a and f_b.

Two traveling waves A and B are described by the equations:

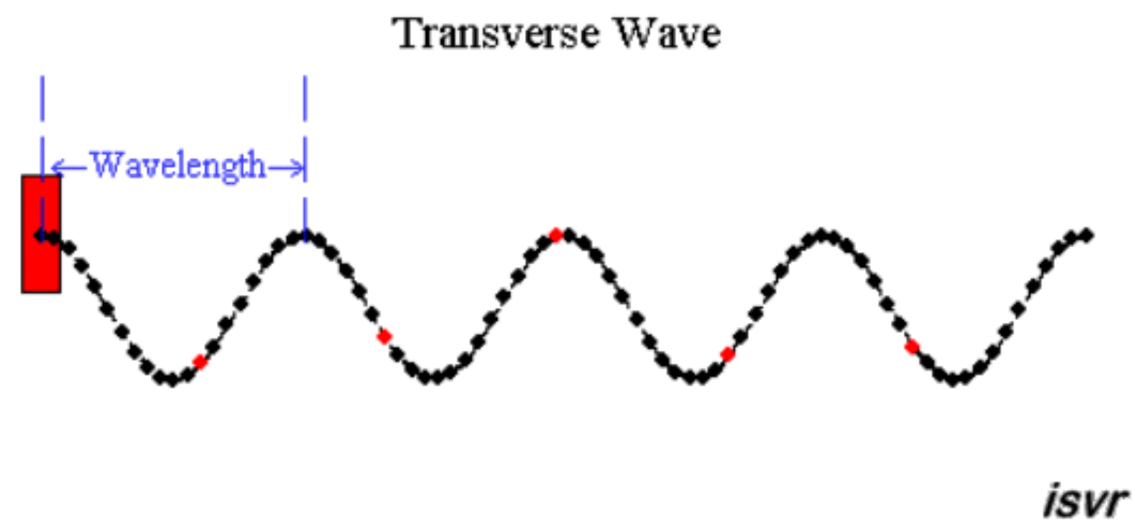
$$y_A(x,t) = (2\,\text{cm})\sin\left[\left(2\,\tfrac{\text{rad}}{\text{m}}\right)x - \left(1\,\tfrac{\text{rad}}{\text{s}}\right)t\right]$$

$$y_B(x,t) = (4\,\text{cm})\sin\left[\left(1\,\tfrac{\text{rad}}{\text{m}}\right)x - \left(0.8\,\tfrac{\text{rad}}{\text{s}}\right)t\right]$$

Which wave has the higher speed?

A) A
B) B
C) Both have the same speed.

What about the up-and-down velocity of a particle in a transverse wave?

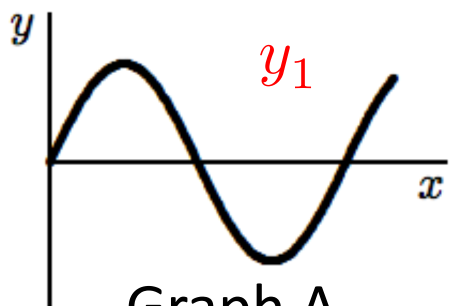


Wave and Particle Velocity Activity

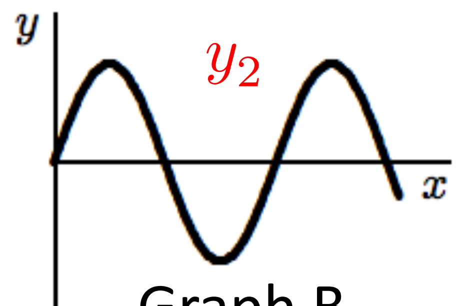
Three traveling sinusoidal waves are on identical strings. The mathematical forms of the waves are shown below. Match each mathematical form to the appropriate graph.

$$y_1(x,t) = Y\sin(3x - 6t) \qquad y_2(x,t) = Y\sin(4x - 8t)$$

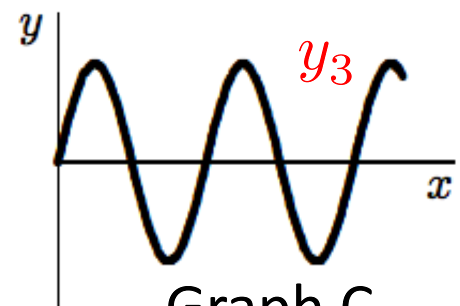
$$y_3(x,t) = Y\sin(6x - 12t)$$



Graph A



Graph B



Graph C

The displacement of a string carrying a traveling sinusoidal wave is given by:

$$y(x,t) = Y\sin(kx - \omega t - \Phi_0)$$

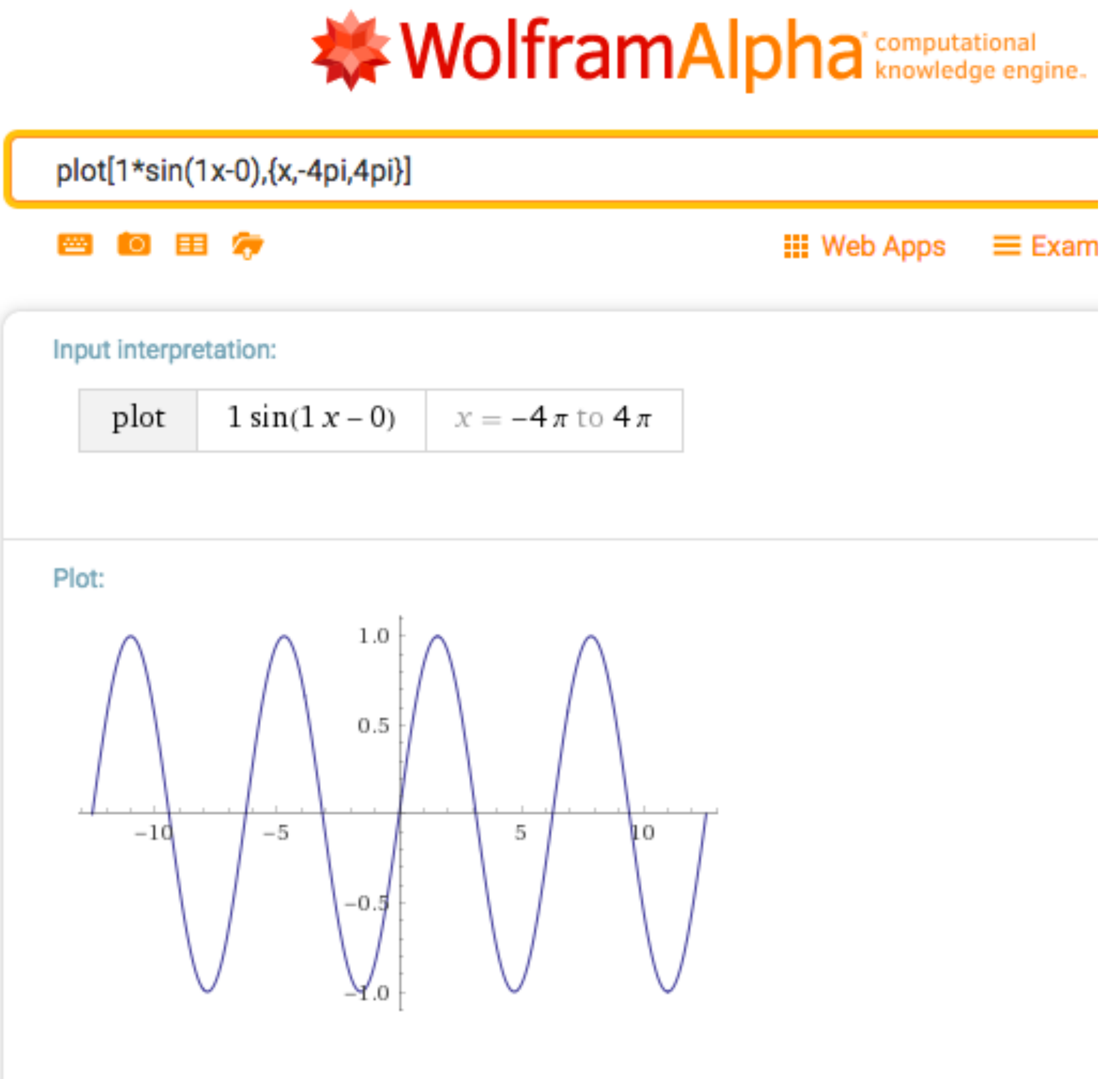
At time $t = 0$ the point $x = 0$ has a velocity of $v = 0$ and positive displacement. The phase constant Φ_0 is:

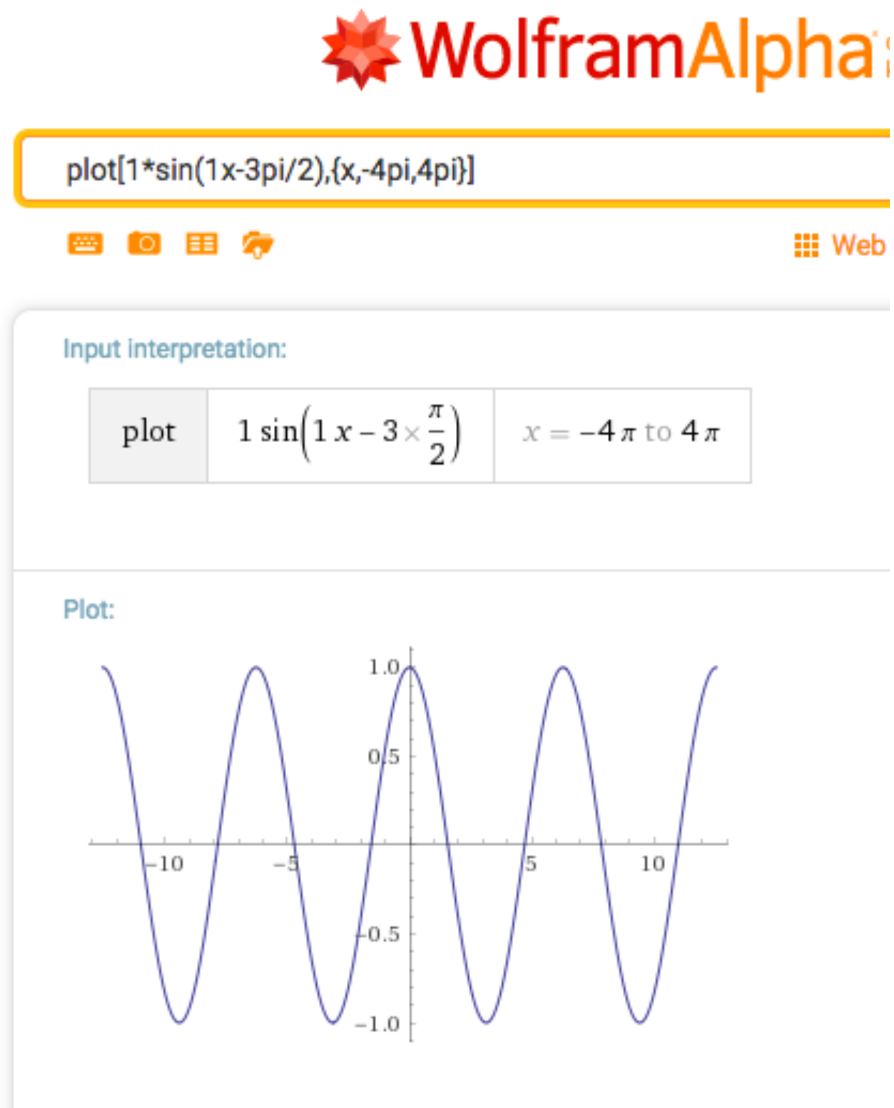
A) $\pi/2$

B) $3\pi/4$

C) π

D) $3\pi/2$


WolframAlpha computational knowledge engine.
plot[1*sin(1x-0),{x,-4pi,4pi}]
Web Apps
Exam
Input interpretation:
plot
1 sin(1 x − 0)
x = −4 π to 4 π
Plot:
1.0
0.5
−10
−5
5
10
−0.5
−1.0


WolframAlpha
plot[1*sin(1x-3pi/2),{x,-4pi,4pi}]
Web
Input interpretation:
plot
1 sin(1 x − 3 × π/2)
x = −4 π to 4 π
Plot:
1.0
0.5
−10
−5
5
10
−0.5
−1.0

Wave Speed Derivation

- What properties of a string (or medium) might determine the wave speed?
- Goal: Use simple physics to find expression for wave speed as function of properties of string.
- (On board)