Homework Set 2

Remember to *present* your solutions to the problem in words. Another student should be able to look at your homework page and be able to figure out what the question was asking without looking at this sheet. please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

1) Where and When?

Two firecrackers explode at the same place in the laboratory and are separated by a time of 12 years.

(a) What is the spatial distance between these two events in a rocket in which the events are separated in time by 13 years?

(b) What is the relative speed of the rocket and laboratory frames? Express your answer as a fraction of the speed of light.

2) Klingon-Federation Treaty

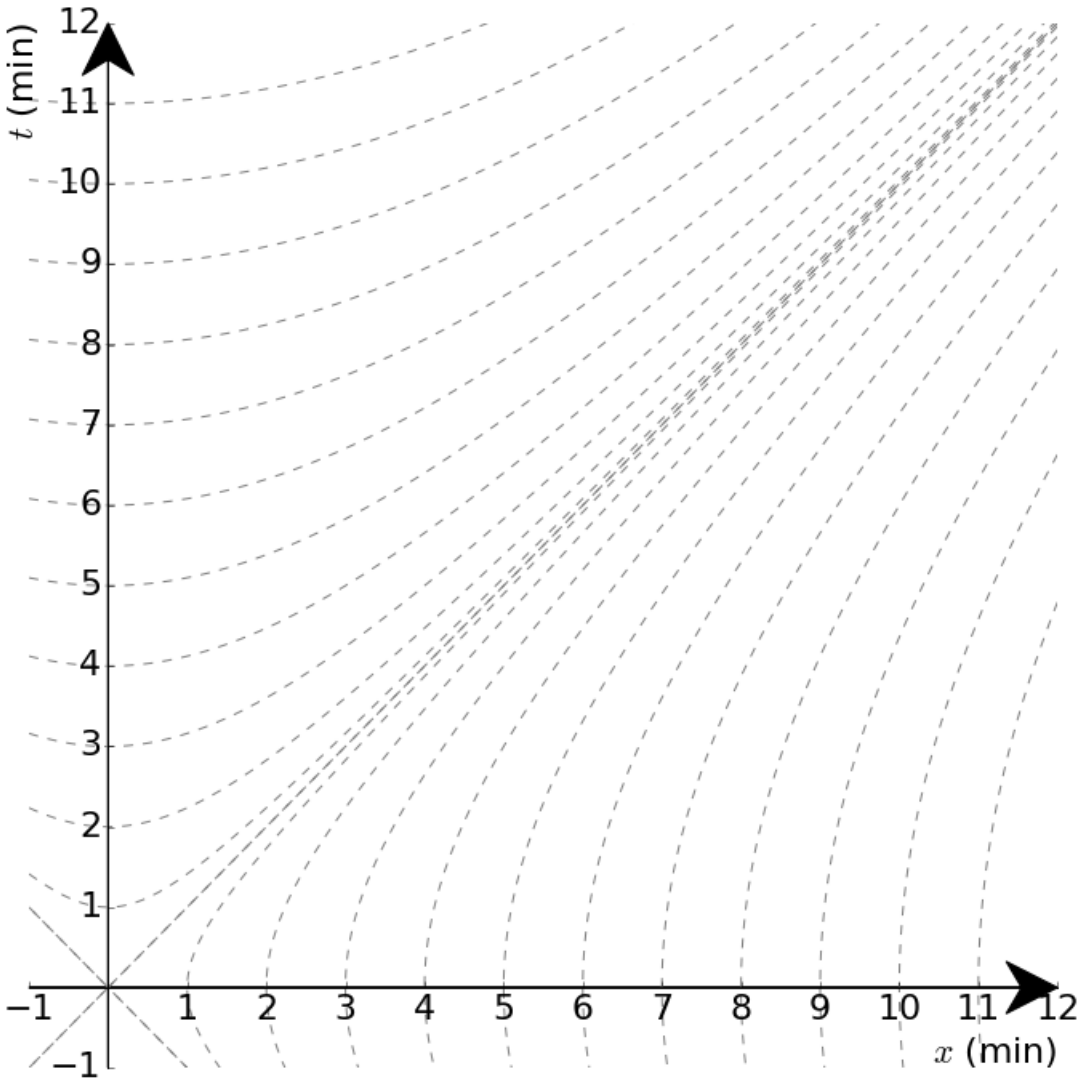
The Federation space cruiser *Execrable* is floating in Federation territory at rest relative to the border of Klingon space, which is 6.0 light-minutes way in the $+x$ direction. Suddenly, a Klingon warship flies past the cruiser in the direction of the border at a speed $\beta = 3/5$. Call this Event A, and let it define time zero in both the Klingon and cruiser reference frames. At $t_B = 5.0$ min according to cruiser clocks, the Klingons emit a parting disrupter blast (event *B*) that travels at the speed of light back to the cruiser. The disrupter blast hits the cruiser and disables it (event *C*) and a bit later (according to cruiser radar measurements) the Klingons cross the border into Klingon territory (event *D*).

(a) Draw a spacetime diagram of the situation, taking the cruiser to define the home frame (S) and the Klingon warship to define the other/moving frame (S′). Draw and label the worldlines of the cruiser, the Klingon territory boundary, the Klingon warship, and the disrupter blast. Draw and label events *A*, *B*, *C*, and *D* as points on your diagram. You may use the blank diagram on the following page or a similarly sized diagram. Use a ruler and protractor.

(b) When does the disrupter blast hit, and when do the Klingons pass into their own territory, according to the clocks in the cruiser's frame? Answer by reading the times of these events directly from the diagram.

(c) The Klingon-Federation Treaty states that it is illegal for a Klingon ship in Federation territory to damage Federation property. When the case comes up in interstellar court, the Klingons claim that they are within the letter of the law: according to measurements made in their reference frame, the damage to the Execrable occurred *after* they had crossed back into Klingon territory: thus they were *not* in Federation territory at the time. Did event *C* (disrupter blast hits the Execrable) *really* happen after event *D* (Klingons cross the border) in the Klingon's frame? Answer this question by using your spacetime diagram.

(d) Check your work with the Lorentz transformation equations.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3) Down with Relativity

Your roommate is at it again. They have the objections to relativity listed below. Respond to each of their objections clearly, decisively, and politely - without criticizing!

(a) "Relativity is preoccupied with how we *observe* things, not with what is *really* happening. Therefore, relativity is not a scientific theory, since science deals with *reality*."

(b) "Relativity offers no way to describe an event without coordinates, and no way to speak about coordinates without referring to one or another particular reference frame. However, physical events have an existence independent of all choice of coordinates and reference frames. Therefore, the special relativity you talk about cannot be the most fundamental theory of events and the relation between events."

4) Kinetic Energy

Find the value of v/c when a particle's kinetic energy equals its rest energy.

5) Mass to Energy

If we could convert rest mass entirely to energy, how much mass would be required to run a 30 W light bulb for a year?

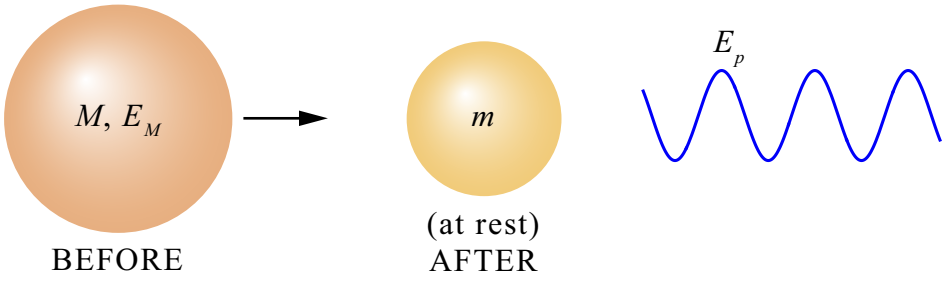
6) A Matter-Antimatter Rocket

When a particle is brought into contact with its corresponding antiparticle, they annihilate each other, converting their rest energy entirely to light energy. A perfect rocket engine might mix antimatter with an equal amount of matter of the same type, and direct the resulting light in a tight beam out of the engine nozzle. No other kind of exhaust could possibly carry more momentum out of the rear of the rocket per unit energy expended than light can.

Imagine a rocket of original mass $M = 90{,}000$ kg sitting at rest in some frame in deep space (M includes the mass of the matter-antimatter fuel). Imagine that it fires its engines, emitting a burst of light having a total (unknown) energy E_L. If after this the ship's final speed is $v = 4/5c$, what is its final mass m?

Hint: you'll need to use two conservation laws to solve this problem. Always justify your use of physical laws!

7) Photon Braking



A radioactive nucleus of known initial mass M and known initial total energy E_M emits a gamma ray (a high-energy photon) in the direction of its motion, drops to its stable nonradioactive state of known mass m, and comes to rest. Photons of light have rest mass equal to zero.

(a) Find an expression for the total energy E_M of the incoming nucleus in terms of M, m, and c. The unknown energy E_p of the outgoing gamma ray should not appear in your expression, nor should the momentum of the incoming nucleus, p_M. *Hints*: You'll need multiple equations to eliminate multiple unknowns! Write equations for conservation of momentum, conservation of energy, and the invariant expression for the particle rest energy in relation to its total energy and kinetic energy. Don't write these expressions using γ_p. Just relate energy, momentum, and mass.

(b) Check your result by examining the limiting cases (*i.e.*, $M = m$, $M >> m$).