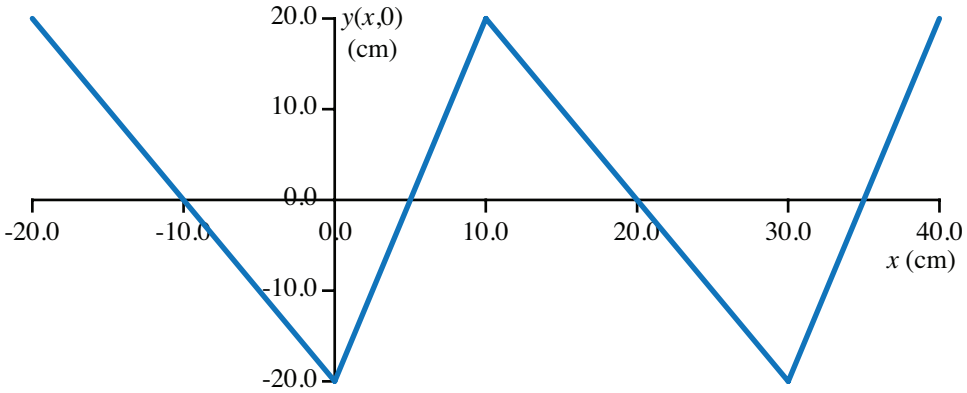
Homework Set 3

Remember to *present* your solutions to the problem in words. Another student should be able to look at your homework page and be able to figure out what the question was asking without looking at this sheet. please show your work and explain your reasoning. Clarity of explanation is as important as mere "correctness of final answer"!

1) Wave on a String I

The figure below represents the profile (t = 0 s) of a transverse wave on a string traveling in the positive x–direction at a speed of 100 cm/s. (The values for x and y are in cm.)



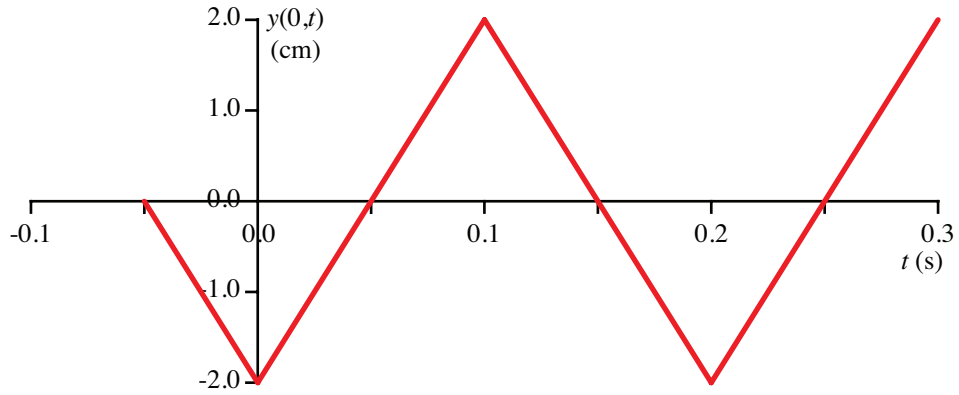
(a) Determine its wavelength.

(b) Notice that as the wave passes any fixed point on the x–axis the string at that location oscillates in time. Draw a graph of y versus t showing how a point on the rope at x = 0 oscillates.

(c) What is the frequency of the wave?

2) Wave on a String II

A transverse wave on a string travels in the negative x–direction at a speed of 40.0 cm/s. The figure below is a graph of y versus t showing how a point on the rope at x = 0 oscillates. (The values for y are in cm.)



(a) Determine the wave's period.

(b) What is the frequency of the wave?

(c) What is the wavelength of the wave?

(d) Sketch the profile of the wave (y versus x)

3) Interpreting a Wavefunction

The wavefunction of a transverse wave on a string is

$$y(x,t) = (10.0)\cos\left[(942.5)x + (15.0)t\right]$$

(a) Oops, I seem to have forgotten to include the units for the constants (bad, bad me).. Rewrite the equation with proper units, assuming I measured all lengths in centimeters and all times in seconds.

(b) What is the frequency of the wave?

(c) What is the wavelength of the wave?

(d) What is the period of the wave?

(e) What is the amplitude of the wave?

(f) What is the propagation constant of the wave, also known as the wavenumber?

(g) What is the phase velocity of the wave?

(h) What is the angular frequency of the wave?

4) Will It Wave?

Determine which of the following describe traveling waves. Where appropriate, draw the profile and find the speed and direction of motion. Feel free to use a computer program of your choice (Excel, Mathematica, WolframAlpha, etc.) for assistance. You may have to express the speed in terms of constants used in the equations.

1) $y(x,t) = e^{-\left(a^2x^2 + b^2t^2 - 2abtx\right)}$

2) $y(z,t) = A\sin\left(az^2 - bt^2\right)$

3) $y(x,t) = A\sin 2\pi\left(\dfrac{x}{a} + \dfrac{t}{b}\right)^2$

4) $y(x,t) = A\cos^2 2\pi(t - x)$

5) The Wave Equation

The wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

is often used to describe the transverse displacement of waves on a stretched string.

(a) Explain the meaning of each of the elements of the equation with reference to the physical string.

(b) Show that the equation for a sinusoidal wave satisfies the wave equation.

6) Superposition Using Complex Exponentials

Two waves of the same amplitude, speed, and frequency travel together in the same region of space. The resultant wave may be written as the sum of individual waves:

$$y(x,t) = A\sin(kx+\omega t) + A\sin(kx-\omega t+\pi)$$

(a) With the help of complex exponentials ($e^{i\theta} = \cos(\theta) + i\sin(\theta)$), show that:

$$y(x,t) = 2A\cos(kx)\sin(\omega t)$$

(b) Describe a physical situation represented by this equation. As part of your description include a sketch and a written description. Indicate what *y* and *x* correspond to in the situation you describe.

(c) How, if at all, would the physical situation you described be different if *k* were twice as large? Explain how you determined your answer.

7) A String Oscillates

A string oscillates according to the equation

$$y(x,t) = (0.50\text{cm})\sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)x\right]\cos\left[\left(40\pi\,\text{s}^{-1}\right)t\right]$$

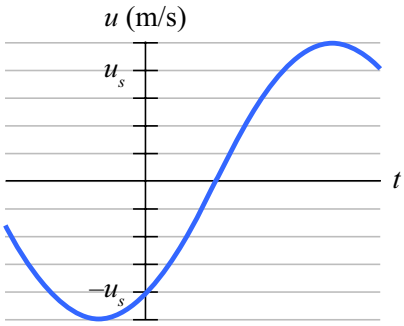
(a) What are the (i) amplitude and (ii) speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation?

(b) What is the distance between nodes?

(c) What is the transverse speed of a particle of the string at the position $x = 1.5\,\text{cm}$ when $t = \frac{9}{8}\,\text{s}$? Interpret the meaning of your results.

8) Finding ϕ_0

The figure to the right shows the transverse velocity *u* vs. *t* on a string at *x* = 0 m, as a wave passes through the string. The scale on the vertical axis is set by u_s = 4.0 m/s. The wave has the form $y(x,t) = Y\sin(kx-\omega t+\phi_0)$. What is ϕ_0 ?



Caution: A calculator does not always give the proper inverse trig function, so check your answer by substituting it and an assumed value of ω into $y(x,t)$ and then plotting the function.