***Homework Set* 6**

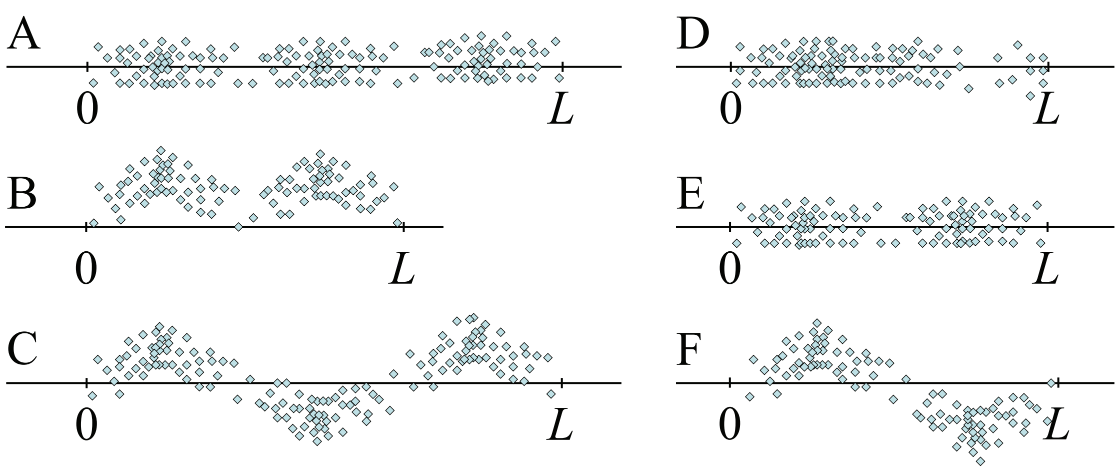
Remember to *present* your solutions to the problem in words. Another student should be able to look at your homework page and be able to figure out what the question was asking without looking at this sheet. please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

1) Electron Wavefunction

Consider the electron wave function

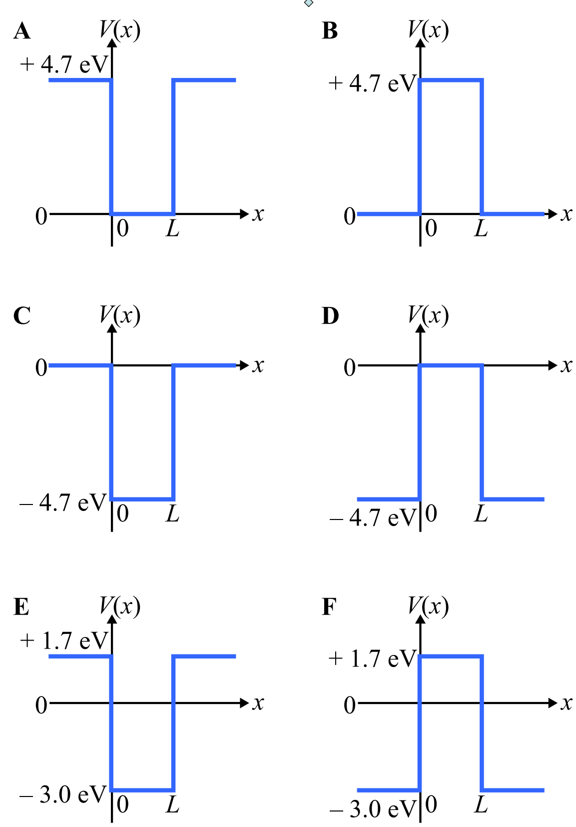
$$\psi(x) = \begin{cases} \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{3\pi}{L}x\right) & 0 \le x \le L \\ 0 & x < 0 \text{ or } x > L \end{cases}$$

If you had a bunch of electrons with this wave function and detected where the electrons were, which pattern below would you expect to see? Explain your reasoning.



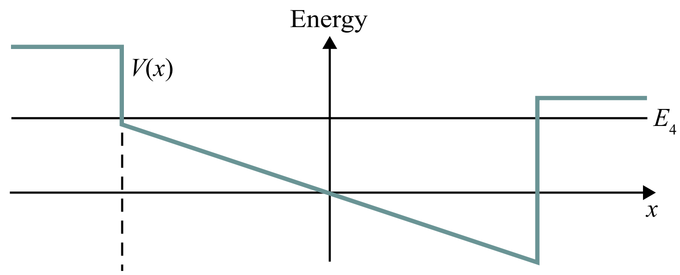
2) Work Function

The work function of copper is – 4.7 eV. Which of the following could be a plot of potential energy vs. position for an electron in a copper wire of length L surrounded by air on either side? (Check all that apply). Explain your reasoning.



3) Draw the Wavefunction

Sketch the energy eigenfunction (solution to the Schrödinger equation) corresponding to the fourth-lowest bound-state energy level of the potential energy function shown below. Indicate the ways in which you have applied our rules for sketching wavefunctions.

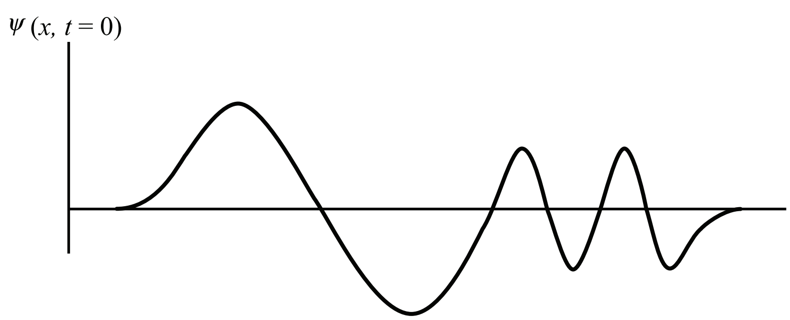


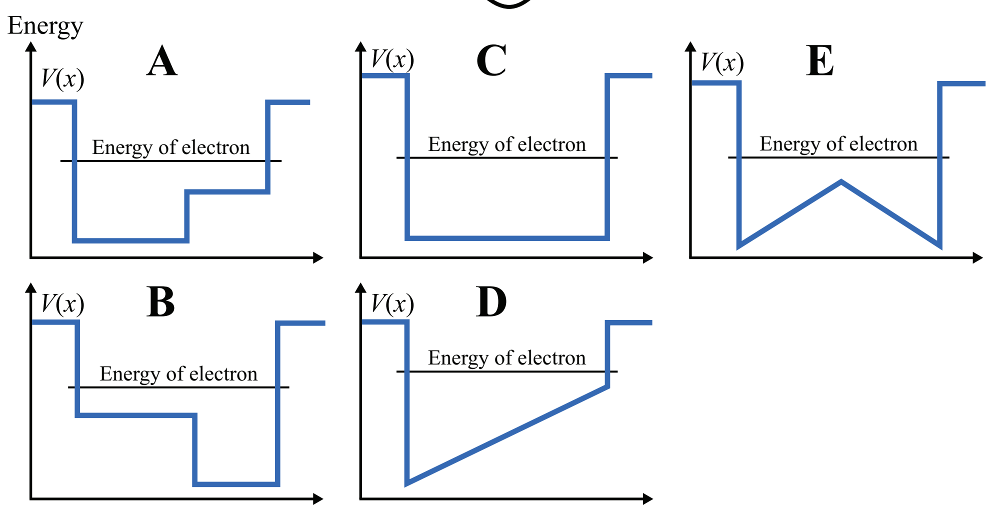
4) Non-Zero Potential

Suppose that in the infinite square well problem, you set the potential energy at the bottom of the well to be equal to some constant V_0, rather than zero. Find an expression for the correct energy levels for the infinite square well of width L with potential energy equal to V_0 at the bottom of the well

5) Potential Well

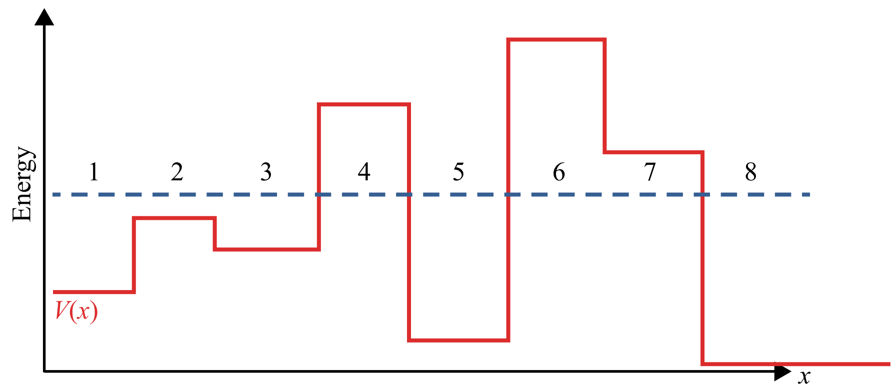
An electron is bound in a potential well. The wave function of the electron is $\Psi(x,t) = \psi(x)e^{-i\omega t}$ where $\psi(x)$ is shown below and is a real function. Which plot of $V(x)$ vs. x below could represent the potential well in which this electron is bound? Explain your reasoning.





6) The Funky Potential

An electron has a total energy as shown by the blue dashed line and a potential energy graph as shown by the red line.



(a) In which regions (1-8), would you expect the solutions to the Schrodinger equation, $\psi(x)$, to be either sines and cosines or complex exponentials? (Check all that apply.) Explain your reasoning.

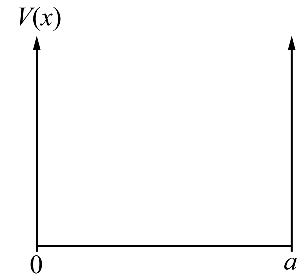
(b) For these regions, rank order them from highest *k* to lowest *k*. Explain your reasoning.

(c) In which regions (1-8), would you expect the solutions to the Schrodinger equation, $\psi(x)$, to be real exponentials? (Check all that apply.) Explain your reasoning.

7) Relating Classical and Quantum Mechanics

Complete the 4 page worksheet titled “Homework: Relating Classical and Quantum Mechanics," which is attached at the end of this assignment. Please print the worksheet and write/draw directly on it.
(From *Tutorials in Physics*: *Quantum Mechanics* – Relating Classical and Quantum Mechanics. McDermott, Heron, Shaffer, and P.E.G., University of Washington)

1. In the tutorial we considered the high-energy limit of a quantum mechanical system. In this problem you will consider the large mass limit.

   Consider the infinite square well potential shown at right.

   a. Sketch the probability density for the ground state and the second excited state ($n = 3$) for a particle of mass m in the infinite square potential shown.

   b. Sketch the probability density for the ground state and the second excited state ($n = 3$) for a particle of mass $10m$ in the same infinite square potential.

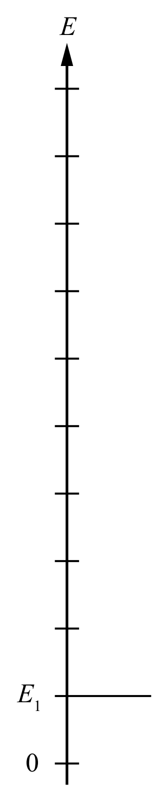
   

   c. On the energy level diagram at right, the ground state of the particle of mass m is labeled E_1.

      Draw the energy levels for each of the following cases.

      - the second excited state of the particle of mass m, E_3
      - the ground state of the particle of mass $10m$, E_1'
      - and the second excited state of the particle of mass $10m$, E_3'

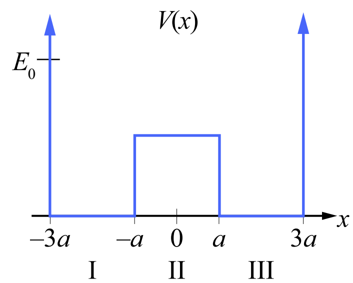
   

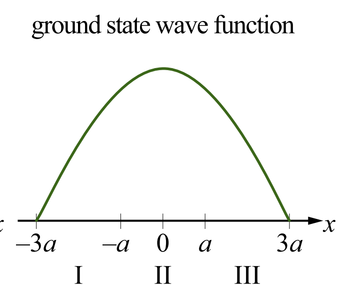
d. Suppose a particle of mass m_1 is in a given excited state n of an infinite square well and particle m_2 (much larger than m_1) is in a different excited state n of the same potential.

   i. Is it possible for the two particles to have the same energy? If so, give a numerical example for which it is possible. In either case, explain.

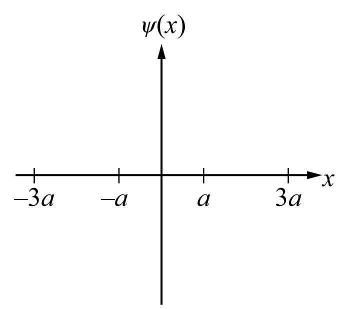
   ii. How do the forms of the wave functions for these two states compare to each other? Which one more closely resembles a classical particle? Explain how your answer is consistent with the correspondence principle.

2. A particle of mass m is bound in the potential well shown at right. The wave function for the ground state is also shown. It has energy E_0.





   a. Sketch the wave function for a very highly-excited energy state of this potential well (*i.e.*, for a state with energy much greater than E_0). Explain your reasoning.



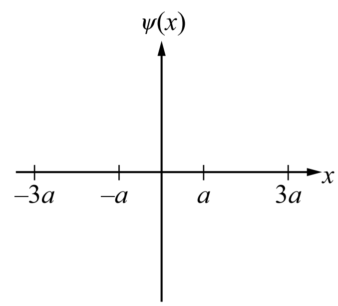
   Explain how your answer is consistent with the correspondence principle.

b. Suppose a very massive particle were bound in an energy eigenstate of this same potential well with energy, E_0, equal to the ground state energy of the **original** particle of mass *m*.

i. In which regions will the wave function have the largest wave number? Explain.

ii. In which regions will the wave function have the largest amplitude? (*Hint*: Use the correspondence principle, and what you learned in the tutorial *Classical Probability*.)

iii. Sketch the wave function for this very massive particle.



c. Compare the wave functions you drew for the highly energetic case and the highly massive case. In particular, consider how the details of the potential well are reflected in both cases.

d. Consider the student discussion below.

Student 1: "You can always tell if you can apply classical reasoning to a quantum mechanics problem by considering the energy alone."

Student 2: "No, you only approach the classical limit if the mass is very large."

Student 3: "The mass alone is not enough to tell. You must consider the particle's energy relative to the energy it has in the ground state."

With which student do you agree? For each student with whom you do not agree, identify a specific case from above which contradicts that student's statement.

3. Consider a particle in the quantum mechanical harmonic oscillator potential.

   a. In the space below, sketch a graph of the wave function and the probability density for a highly-excited energy eigenstate in this potential.

   b. On your graphs above, identify the regions where the particle is most likely and least likely to be found. Explain.

   c. On your graphs above, identify the regions where the wave number of the particle is largest and smallest. Explain.

   d. The classical probability distribution for a particle in a harmonic oscillator potential has the form $\rho(x) = \dfrac{1}{\pi\sqrt{L^2 - x^2}}$, where L is the maximum displacement. Graph this in the space below and compare it to your result from part a.

   e. Are your answers consistent with the correspondence principle? Explain.

   f. For the quantum particle, is the probability that the particle is found to be beyond the classical turning point equal to zero? Explain.

   If necessary, modify your graphs so that they are consistent with your answer.