Remember, you may be called upon to present any one of these problems to the class, so you need to bring your complete assignment, where you have shown your work and explained your reasoning. Presentations do not receive full credit for a correct final answer without explanation or derivation.

1) Momentum

Show that:

$$\frac{d\langle p\rangle}{dt} = \left\langle -\frac{\partial V}{\partial x}\right\rangle \tag{1}$$

Is this relationship consistent with classical physics?

This problem gives us practice with using operators and manipulating Ψ. The method here will be similar to our in-class proofs showing that normalization remains constant for all time, and how we derived the momentum operator. Apply product rule, use the Schrodinger equation to solve for

$$\frac{\partial \Psi}{\partial t} \text{ and } \frac{\partial \Psi^*}{\partial t}$$

and many terms will go to zero due to the wavefunction going to zero as x approaches$\pm\infty$. Recall also that

$$\frac{\partial^2 \Psi}{\partial x \partial t} = \frac{\partial^2 \Psi}{\partial t \partial x}$$

2) Wavefunction 1

Consider a particle with a wave function:

$$\Psi(x,t) = \begin{cases} Ae^{-iEt/\hbar}(4-(x/a)^2), & |x| \leq 2a \\ 0, & |x| > 2a \end{cases}$$

where *a* is a positive, real constant.

Some hints: Always be on the lookout for odd integrals! And, if an integral is EVEN, it might be advantageous to change your limits of integration... for example, instead of integrating from –2a to 2a, just take double the integral from 0 to 2a.

(a) Find $|\Psi(x,t)|^2$. Is it a function of x and t, or a function of x only?

(b) Normalize $\Psi(x,t)$. (This means to find A. Show that the answer is $A = \sqrt{\frac{15}{512a}}$)

(c) Sketch $|\Psi(x,t)|^2$ as a function of x. Your sketch doesn't have to be perfect, but try to get the basic shape, width, and positioning correct.

(d) Compute the expectation value $\langle x\rangle$.

(e) Compute $\langle x^2\rangle$. *Answer*: $\frac{4}{7}a^2$

(f) Compute σ_x.

(g) Compare your calculation results to the sketch you made earlier. Explain how your calculations agree, or fail to agree, with your sketch.

This and the next problem allow us to practice many of the things we can "do" with a wavefunction, once we have it. We still haven't talked about how to find the wavefunction, and that will be the goal of Chapter 2. You should be able to take integrals of polynomials and add fractions by hand, but it's okay to check your answers with Mathematica.

3) Wavefunction 2

(a) Find $\langle p \rangle$ for the wave function in the previous problem.

$$\Psi(x,t) = \begin{cases} Ae^{-iEt/\hbar}(4-(x/a)^2), & |x| \leq 2a \\ 0, & |x| > 2a \end{cases}$$

(b) Find $\langle p^2 \rangle$ for the save wave function. *Answer*: $\frac{5}{8}\frac{\hbar^2}{a^2}$

(c) Confirm that the units for $\langle p^2 \rangle$ are correct.

(d) Calculate $\sigma_p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$.

(e) Find the product $\sigma_x \sigma_p$ for this wave function (using your result forσ_x from the previous problem). Show that it is consistent with Heisenberg's "Uncertainty Principle": $\sigma_x \sigma_p \geq \hbar/2$

4) Wave Equation

The second-order differential equation,

$$\frac{d^2 f(x)}{dx^2} = -k^2 f(x) \tag{2}$$

has two linearly independent solutions. These can be written in more than one way, and two convenient forms are

$$f(x) = Ae^{ikx} + Be^{-ikx} \tag{3}$$

$$f(x) = a\sin(kx) + b\cos(kx) \tag{4}$$

Verify that both are solutions of the equation (2) above. Since both are equally good solutions, we must be able to determine a and b in terms of A and B; do so.

Consider this problem a warm-up to lead us into Chapter 2. You're relying on your math skills; there's no physics here (*yet*).