3.7 FINAL THOUGHTS: WHAT THE STATE FUNCTION ISN'T

As if by magic, Postulates I and II have transformed de Broglie's vague notion of a matter wave into a beautiful mathematical construct: the state function. The Born Interpretation of this function, the bulwark of quantum theory, defines the very character of that theory as intrinsically probabilistic, statistical—and limited. From state functions, we can calculate probabilities for various outcomes of measurements and such statistical quantities as the average value and the uncertainty. But nothing more.

Even so simple a question as "Where is a particular electron" is beyond the purview of quantum mechanics. At most, a quantum physicist can determine the probability of finding *one of an ensemble* of electrons at a particular location, the mean value of the positions obtained in an ensemble measurement, the position where an electron is most likely to be found, and a rough measure of the size of the region where most of the electrons will appear. In this chapter we've begun to learn how to do such calculations.

Other, more philosophical questions also are beyond quantum mechanics. For example, in this chapter I have talked a lot about Ψ but have not addressed questions such as: what, precisely, is this function? Where does it exist? In ordinary, geometrical space, like an electromagnetic wave? And what is the nature of its connection to the material system whose state it represents? I've dodged such questions because they lie on the fringes of orthodox quantum theory, in the domain of speculations, rather than answers. But the human mind abhors a vacuum; we seem to need to relate important new ideas to familiar concepts, and nearly every newcomer to quantum mechanics must engage in the struggle to get rid of classically based answers to these questions.

Our tendency to draw on classical physics as we grope for an understanding of quantum theory is certainly understandable. We're seduced by similarities. For example, the state function plays a role in quantum mechanics that is *analogous* to that played by the trajectory in classical mechanics: it is the basic mathematical element, the state descriptor, of the theory. But the identification must end with this analogue! A wave function is not a trajectory. My point, which you'll hear me rant about again, is that at both the conceptual and the mathematical level, quantum mechanics is not just a funny-looking reformulation of classical physics. The two physical theories are fundamentally, physically different.

Even words lead us astray. When thinking about a "wave function", we naturally tend to envision a wave in the classical sense: an actual, extant, three-dimensional waveform that propagates and carries energy. To be sure, we know from Chap. 2 that to picture a microscopic particle as a wave is wrong. But what of the "wave functions" that represent states of these particles? They seem rather like classical waves, don't they?

They do—but they aren't. Although the *properties* and *behavior* of quantum waves are familiar, these waves are totally unlike any you have ever studied. For example, unlike classical waves, *quantum waves don't carry energy.* So, it's impossible to detect wave functions *per se*; these bizarre entities manifest themselves only indirectly, through their influence on the behavior of quantum particles as they propagate through space. (This feature led Einstein to refer to quantum waves as "ghost waves".) The foundation of your thinking about wave functions should be not the classical concept of a wave but rather the

novel notion that *state functions carry information*—probabilistic and statistical information about how the particles whose states they represent would behave were their properties measured.

Moreover, wave functions do not exist in space, at least not in the sense that classical waves exist. Consider space. The space in which we walk around, frolic, study quantum mechanics, and perform experiments is called $\mathfrak{R}^3$; we specify locations in $\mathfrak{R}^3$ by the values of three real numbers in an orthogonal coordinate system.

Now consider a state function. Such a function is complex and so could not exist in $\mathfrak{R}^3$, for only real numbers and functions exist in this space. Moreover, the state function for a many-particle system depends on several coordinates. How could a function such as $\Psi(\mathbf{r_1}, \mathbf{r_2}, t)$ exist in geometrical space? It's nonsense to suggest that such a function exists, as does an electromagnetic wave, in a space of three spatial coordinates.

The moral of these ruminations is that whatever Ψ is, it's not a conventional undulation in space, like an electric field. So where does this leave us in our quest to understand the reality behind the state function? Do wave functions really exist? Here is Werner Heisenberg trying to answer this question.

> The probability wave … [stands] in the middle between the idea of an event and the actual event, a strange kind of physical reality just in the middle between *possibility* and *reality*. [emphasis mine]

Heisenberg's answer is strange indeed, for it leaves us in a kind of metaphysical limbo without an experimental escape route. When we're not looking at microscopic particles, they behave in ways that correspond to the classical wave model; yet, when we measure their properties, they behave like classical particles. The postulates of this chapter provide a way out of this logical conundrum: we assume that complex functions exist that are somehow associated with the states of a microscopic system. These functions carry probabilistic information about the outcomes of measurements of the observables of the system. Finally, they can interfere according to the Principle of Superposition—and therein lies the explanation of the observed behavior of quantum particles.

As far as anyone knows, the behavior of all microscopic particles is governed by probability waves: every electron, proton, and neutron in every atom in every blob of matter in the universe (including your author) is guided by its state function. As Nick Herbert says in his *Quantum Reality* (Doubleday/Anchor Press, 1985):

> In each atom … something seems smeared out to fill the atom, an indescribable something we call the "probability cloud", "realm of positional possibilities", "electron wave function", or "quantum stuff" without really being very sure what we're talking about. Whatever it is, though, the whole world is made of it.

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Understanding Quantum Physics
(Englewood Cliffs, New Jersey: Prentice-Hall, 1990)