The following tables are taken from the Mathematical Section of the Engineers' Manual in which are consolidated the principal formulas and tables of Mathematics, Mechanics, Hydraulics, Heat and Electricity. The complete Mathematical Section is published under the title, A Manual of Mathematics.

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A TABLE OF INTEGRALS

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TABLE OF DERIVATIVES

Functions of x are represented by u and v, constants are represented by a, n, and e.

$$\frac{d}{dx}(x) = 1.$$

$$\frac{d}{dx}(a) = 0.$$

$$\frac{d}{dx}(u \pm v \pm \cdots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \cdots.$$

$$\frac{d}{dx}(au) = a\frac{du}{dx}.$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}.$$

$$\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx}.$$

$$\frac{d}{dx}\ln u = \frac{1}{u}\frac{du}{dx}.$$

$$\frac{d}{dx}a^u = a^u \ln a\frac{du}{dx}.$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}.$$

$$\frac{d}{dx}u^v = vu^{v-1}\frac{du}{dx} + u^v \ln u\frac{dv}{dx}.$$

$$\frac{d}{dx}\sin u = \cos u\frac{du}{dx}.$$

$$\frac{d}{dx}\cos u = -\sin u\frac{du}{dx}.$$

$$\frac{d}{dx}\tan u = \sec^2 u\frac{du}{dx}.$$

$$\frac{d}{dx}\cot u = -\csc^2 u\frac{du}{dx}.$$

$$\frac{d}{dx}\sec u = \sec u \tan u\frac{du}{dx}.$$

$$\frac{d}{dx}\csc u = -\csc u \cot u\frac{du}{dx}.$$

$$\frac{d}{dx}\text{vers}\, u = \sin u\frac{du}{dx}.$$

$$\frac{d}{dx}\sin^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \quad \left(\text{where } \sin^{-1} u \text{ lies between } -\frac{\pi}{2} \text{ and } +\frac{\pi}{2}\right).$$

$$\frac{d}{dx}\cos^{-1} u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \quad (\text{where } \cos^{-1} u \text{ lies between } 0 \text{ and } \pi).$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}.$$

$$\frac{d}{dx}\cot^{-1} u = -\frac{1}{1+u^2}\frac{du}{dx}.$$

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx} \quad (\text{where } \sec^{-1} u \text{ lies between } 0 \text{ and } \pi).$$

$$\frac{d}{dx}\csc^{-1} u = -\frac{1}{u\sqrt{u^2-1}}\frac{du}{dx} \quad \left(\text{where } \csc^{-1} u \text{ lies between } -\frac{\pi}{2} \text{ and } +\frac{\pi}{2}\right).$$

$$\frac{d}{dx}\text{vers}^{-1} u = \frac{1}{\sqrt{2u-u^2}}\frac{du}{dx} \quad (\text{where } \text{vers}^{-1} u \text{ lies between } 0 \text{ and } \pi).$$

TABLE OF INTEGRALS

Fundamental Forms

$$\int df(x) = f(x) + C.$$

$$d\int f(x)\, dx = f(x)\, dx.$$

$$\int [f_1(x) \pm f_2(x) \pm \cdots]\, dx = \int f_1(x)\, dx \pm \int f_2(x)\, dx \pm \cdots.$$

$$\int a f(x)\, dx = a\int f(x)\, dx, \text{ where } a \text{ is any constant.}$$

$$\int u^n\, du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1); \text{ u is any function of x.}$$

$$\int \frac{du}{u} = \ln u + C; \text{ u is any function of x.}$$

$$\int u\, dv = uv - \int v\, du; \text{ u and v are any functions of x.}$$

NOTE. In the following table, the constant of integration (C) is omitted but should be added to the result of every integration. The letter x represents any variable; the letter u represents any function of x; all other letters represent constants which may have any finite value unless otherwise indicated; $\ln = \log_e$; all angles are in radians.

Functions containing ax + b

1 $\int (ax+b)^n\, dx = \frac{1}{a(n+1)}(ax+b)^{n+1}. \quad (n \neq -1)$

2 $\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b).$

3 $\int x(ax+b)^n\, dx = \frac{1}{a^2(n+2)}(ax+b)^{n+2} - \frac{b}{a^2(n+1)}(ax+b)^{n+1}.$ $(n \neq -1, -2)$

4 $\int \frac{x\, dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b).$

5 $\int \frac{x\, dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b).$

6 $\int x^2(ax+b)^n\, dx = \frac{1}{a^3}\left[\frac{(ax+b)^{n+3}}{n+3} - 2b\frac{(ax+b)^{n+2}}{n+2} + b^2\frac{(ax+b)^{n+1}}{n+1}\right].$ $(n \neq -1, -2, -3)$

7 $\int \frac{x^2\, dx}{ax+b} = \frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2 - 2b(ax+b) + b^2\ln(ax+b)\right].$

8 $\int \frac{x^2\, dx}{(ax+b)^2} = \frac{1}{a^3}\left[(ax+b) - 2b\ln(ax+b) - \frac{b^2}{ax+b}\right].$

9 $\int \frac{x^2\, dx}{(ax+b)^3} = \frac{1}{a^3}\left[\ln(ax+b) + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right].$

10 $\int x^m (ax+b)^n \, dx$

$$= \frac{1}{a(m+n+1)} \left[x^m (ax+b)^{n+1} - mb \int x^{m-1} (ax+b)^n \, dx \right]$$

$$= \frac{1}{m+n+1} \left[x^{m+1} (ax+b)^n + nb \int x^m (ax+b)^{n-1} dx \right]. \quad \begin{pmatrix} m \text{ pos.} \\ m+n \\ +1 \neq 0 \end{pmatrix}$$

11 $\int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln \frac{x}{ax+b}.$

12 $\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2} \ln \frac{ax+b}{x}.$

13 $\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} - \frac{1}{b^2} \ln \frac{ax+b}{x}.$

14 $\int \frac{dx}{x^2(ax+b)^2} = -\frac{b+2ax}{b^2 x(ax+b)} + \frac{2a}{b^3} \ln \frac{ax+b}{x}.$

15 $\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \ln \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}.$ (b pos.)

16 $\int \frac{dx}{x\sqrt{ax+b}} = \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{ax+b}{-b}}.$ (b neg.)

17 $\int \frac{dx}{x(ax+b)^{\frac{n}{2}}} = \frac{2}{b(n-2)(ax+b)^{\frac{n}{2}-1}} + \frac{1}{b} \int \frac{dx}{x(ax+b)^{\frac{n}{2}-1}}.$ (n odd and pos.)

18 $\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + \sqrt{b} \ln \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}.$ (b pos.)

19 $\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} - 2\sqrt{-b} \tan^{-1} \sqrt{\frac{ax+b}{-b}}.$ (b neg.)

20 $\int \frac{(ax+b)^{\frac{n}{2}}}{x} dx = \frac{2}{n} (ax+b)^{\frac{n}{2}} + b \int \frac{(ax+b)^{\frac{n}{2}-1}}{x} dx.$ (n odd and pos.)

21 $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b\sqrt{b}} \ln \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}.$ (b pos.)

22 $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{b\sqrt{-b}} \tan^{-1} \sqrt{\frac{ax+b}{-b}}.$ (b neg.)

23 $\int \frac{dx}{(ax+b)(px+q)} = \frac{1}{bp-aq} \ln \frac{px+q}{ax+b}.$ $(bp-aq \neq 0)$

24 $\int \frac{dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq} \left[\frac{1}{ax+b} + \frac{p}{bp-aq} \ln \frac{px+q}{ax+b} \right].$ $(bp-aq \neq 0)$

25 $\int \frac{dx}{(ax+b)^n (px+q)^m} = \frac{1}{(m-1)(bp-aq)} \left[\frac{1}{(ax+b)^{n-1}(px+q)^{m-1}} \right.$

$$\left. - a(m+n-2) \int \frac{dx}{(ax+b)^n (px+q)^{m-1}} \right].$$

$(m > 1, n \text{ pos.}, bp-aq \neq 0)$

26 $\int \frac{x\,dx}{(ax+b)(px+q)} = \frac{1}{bp-aq} \left[\frac{b}{a} \ln(ax+b) - \frac{q}{p} \ln(px+q) \right].$ $(bp-aq \neq 0)$

27 $\int \frac{x\,dx}{(ax+b)^2(px+q)} = \frac{1}{bp-aq} \left[-\frac{b}{a(ax+b)} - \frac{q}{bp-aq} \ln \frac{px+q}{ax+b} \right].$

$(bp-aq \neq 0)$

28 $\int \frac{px+q}{\sqrt{ax+b}} dx = \frac{2}{3a^2} (3aq - 2bp + apx) \sqrt{ax+b}.$

29 $\int \frac{\sqrt{ax+b}}{px+q} dx = \frac{2\sqrt{ax+b}}{p} - \frac{2}{p} \sqrt{\frac{aq-bp}{p}} \tan^{-1} \sqrt{\frac{p(ax+b)}{aq-bp}}.$

(p pos., $aq > bp$)

30 $\int \frac{\sqrt{ax+b}}{px+q} dx = \frac{2\sqrt{ax+b}}{p} + \frac{1}{p} \sqrt{\frac{bp-aq}{p}} \ln \frac{\sqrt{p(ax+b)} - \sqrt{bp-aq}}{\sqrt{p(ax+b)} + \sqrt{bp-aq}}.$

(p pos., $bp > aq$)

31 $\int \frac{dx}{(px+q)\sqrt{ax+b}} = \frac{2}{\sqrt{p}\sqrt{aq-bp}} \tan^{-1} \sqrt{\frac{p(ax+b)}{aq-bp}}.$ (p pos., $aq > bp$)

32 $\int \frac{dx}{(px+q)\sqrt{ax+b}} = -\frac{1}{\sqrt{p}\sqrt{bp-aq}} \ln \frac{\sqrt{p(ax+b)} - \sqrt{bp-aq}}{\sqrt{p(ax+b)} + \sqrt{bp-aq}}.$

(p pos., $bp > aq$)

33 $\int \frac{\sqrt{px+q}}{\sqrt{ax+b}} dx = \frac{1}{a} \sqrt{(ax+b)} \sqrt{(px+q)} - \frac{bp-aq}{a\sqrt{ap}}$

$$\ln \left(\sqrt{p(ax+b)} + \sqrt{a(px+q)} \right).$$ (a and p, same sign)

$$= \frac{1}{a} \sqrt{(ax+b)} \sqrt{(px+q)} - \frac{bp-aq}{a\sqrt{-ap}} \tan^{-1} \frac{\sqrt{-ap(ax+b)}}{a\sqrt{px+q}}$$

(a and p have opposite signs)

$$= \frac{1}{a} \sqrt{(ax+b)} \sqrt{px+q} + \frac{bp-aq}{2a\sqrt{-ap}}$$

$$\sin^{-1} \frac{2apx+aq+bp}{bp-aq}.$$ (a and p have opposite signs)

Functions containing $ax^2 + b$

34 $\int \frac{dx}{ax^2+b} = \frac{1}{\sqrt{ab}} \tan^{-1} \left(x\sqrt{\frac{a}{b}} \right).$ (a and b pos.)

35 $\int \frac{dx}{ax^2+b} = \frac{1}{2\sqrt{-ab}} \ln \frac{x\sqrt{a} - \sqrt{-b}}{x\sqrt{a} + \sqrt{-b}}.$ (a pos., b neg.)

$$= \frac{1}{2\sqrt{-ab}} \ln \frac{\sqrt{b} + x\sqrt{-a}}{\sqrt{b} - x\sqrt{-a}}.$$ (a neg., b pos.)

36 $\int \frac{dx}{(ax^2+b)^n} = \frac{1}{2(n-1)b} \frac{x}{(ax^2+b)^{n-1}} + \frac{2n-3}{2(n-1)b} \int \frac{dx}{(ax^2+b)^{n-1}}$ $\begin{pmatrix} n \text{ integ.} \\ > 1 \end{pmatrix}$

37 $\int (ax^2+b)^n x\,dx = \frac{1}{2a} \frac{(ax^2+b)^{n+1}}{n+1}.$ $(n \neq -1)$

38 $\int \frac{x\,dx}{ax^2+b} = \frac{1}{2a} \ln(ax^2+b).$

39 $\int \frac{dx}{x(ax^2+b)} = \frac{1}{2b} \ln \frac{x^2}{ax^2+b}.$

40 $\int \frac{x^2\,dx}{ax^2+b} = \frac{x}{a} - \frac{b}{a} \int \frac{dx}{ax^2+b}.$

41 $$\int \frac{x^2\,dx}{(ax^2+b)^n} = -\frac{1}{2(n-1)a}\,\frac{x}{(ax^2+b)^{n-1}} + \frac{1}{2(n-1)a}\int \frac{dx}{(ax^2+b)^{n-1}}.$$ (n integ. > 1)

42 $$\int \frac{dx}{x^2(ax^2+b)^n} = \frac{1}{b}\int \frac{dx}{x^2(ax^2+b)^{n-1}} - \frac{a}{b}\int \frac{dx}{(ax^2+b)^n}.$$ (n pos. integ.)

43 $$\int \sqrt{ax^2+b}\,dx = \frac{x}{2}\sqrt{ax^2+b} + \frac{b}{2\sqrt{a}}\ln\left(x\sqrt{a}+\sqrt{ax^2+b}\right).$$ (a pos.)

44 $$\int \sqrt{ax^2+b}\,dx = \frac{x}{2}\sqrt{ax^2+b} + \frac{b}{2\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right).$$ (a neg.)

45 $$\int \frac{dx}{\sqrt{ax^2+b}} = \frac{1}{\sqrt{a}}\ln\left(x\sqrt{a}+\sqrt{ax^2+b}\right).$$ (a pos.)

46 $$\int \frac{dx}{\sqrt{ax^2+b}} = \frac{1}{\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right).$$ (a neg.)

47 $$\int \sqrt{ax^2+b}\,x\,dx = \frac{1}{3a}(ax^2+b)^{\frac{3}{2}}.$$

48 $$\int \frac{x\,dx}{\sqrt{ax^2+b}} = \frac{1}{a}\sqrt{ax^2+b}.$$

49 $$\int \frac{\sqrt{ax^2+b}}{x}\,dx = \sqrt{ax^2+b} + \sqrt{b}\ln\frac{\sqrt{ax^2+b}-\sqrt{b}}{x}.$$ (b pos.)

50 $$\int \frac{\sqrt{ax^2+b}}{x}\,dx = \sqrt{ax^2+b} - \sqrt{-b}\tan^{-1}\frac{\sqrt{ax^2+b}}{\sqrt{-b}}.$$ (b neg.)

51 $$\int \frac{dx}{x\sqrt{ax^2+b}} = \frac{1}{\sqrt{b}}\ln\frac{\sqrt{ax^2+b}-\sqrt{b}}{x}.$$ (b pos.)

52 $$\int \frac{dx}{x\sqrt{ax^2+b}} = \frac{1}{\sqrt{-b}}\sec^{-1}\left(x\sqrt{-\frac{a}{b}}\right).$$ (b neg.)

53 $$\int \sqrt{ax^2+b}\,x^2\,dx = \frac{x}{4a}(ax^2+b)^{\frac{3}{2}} - \frac{bx}{8a}\sqrt{ax^2+b} - \frac{b^2}{8a\sqrt{a}}\ln\left(x\sqrt{a}+\sqrt{ax^2+b}\right).$$ (a pos.)

54 $$\int \sqrt{ax^2+b}\,x^2\,dx = \frac{x}{4a}(ax^2+b)^{\frac{3}{2}} - \frac{bx}{8a}\sqrt{ax^2+b} - \frac{b^2}{8a\sqrt{-a}}\sin^{-1}\left(x\sqrt{\frac{-a}{b}}\right).$$ (a neg.)

55 $$\int \frac{x^2\,dx}{\sqrt{ax^2+b}} = \frac{x}{2a}\sqrt{ax^2+b} - \frac{b}{2a\sqrt{a}}\ln\left(x\sqrt{a}+\sqrt{ax^2+b}\right).$$ (a pos.)

56 $$\int \frac{x^2\,dx}{\sqrt{ax^2+b}} = \frac{x}{2a}\sqrt{ax^2+b} - \frac{b}{2a\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right).$$ (a neg.)

57 $$\int \frac{\sqrt{ax^2+b}}{x^2}\,dx = -\frac{\sqrt{ax^2+b}}{x} + \sqrt{a}\ln\left(x\sqrt{a}+\sqrt{ax^2+b}\right).$$ (a pos.)

58 $$\int \frac{\sqrt{ax^2+b}}{x^2}\,dx = -\frac{\sqrt{ax^2+b}}{x} - \sqrt{-a}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right).$$ (a neg.)

59 $$\int \frac{dx}{x^2\sqrt{ax^2+b}} = -\frac{\sqrt{ax^2+b}}{bx}.$$

60 $$\int \frac{x^n\,dx}{\sqrt{ax^2+b}} = \frac{x^{n-1}\sqrt{ax^2+b}}{na} - \frac{(n-1)\,b}{na}\int \frac{x^{n-2}\,dx}{\sqrt{ax^2+b}}.$$ (n pos.)

61 $$\int x^n\sqrt{ax^2+b}\,dx = \frac{x^{n-1}(ax^2+b)^{\frac{3}{2}}}{(n+2)\,a} - \frac{(n-1)\,b}{(n+2)\,a}\int x^{n-2}\sqrt{ax^2+b}\,dx.$$ (n pos.)

62 $$\int \frac{\sqrt{ax^2+b}\,dx}{x^n} = -\frac{(ax^2+b)^{\frac{3}{2}}}{b\,(n-1)\,x^{n-1}} - \frac{(n-4)\,a}{(n-1)\,b}\int \frac{\sqrt{ax^2+b}}{x^{n-2}}\,dx.$$ (n > 1)

63 $$\int \frac{dx}{x^n\sqrt{ax^2+b}} = -\frac{\sqrt{ax^2+b}}{b\,(n-1)\,x^{n-1}} - \frac{(n-2)\,a}{(n-1)\,b}\int \frac{dx}{x^{n-2}\sqrt{ax^2+b}}.$$ (n > 1)

64 $$\int (ax^2+b)^{\frac{3}{2}}\,dx = \frac{x}{8}(2\,ax^2+5\,b)\sqrt{ax^2+b} + \frac{3\,b^2}{8\sqrt{a}}\ln\left(x\sqrt{a}+\sqrt{ax^2+b}\right).$$ (a pos.)

65 $$\int (ax^2+b)^{\frac{3}{2}}\,dx = \frac{x}{8}(2\,ax^2+5\,b)\sqrt{ax^2+b} + \frac{3\,b^2}{8\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right).$$ (a neg.)

66 $$\int \frac{dx}{(ax^2+b)^{\frac{3}{2}}} = \frac{x}{b\sqrt{ax^2+b}}.$$

67 $$\int (ax^2+b)^{\frac{3}{2}}\,x\,dx = \frac{1}{5\,a}(ax^2+b)^{\frac{5}{2}}.$$

68 $$\int \frac{x\,dx}{(ax^2+b)^{\frac{3}{2}}} = -\frac{1}{a\sqrt{ax^2+b}}.$$

69 $$\int \frac{x^2\,dx}{(ax^2+b)^{\frac{3}{2}}} = -\frac{x}{a\sqrt{ax^2+b}} + \frac{1}{a\sqrt{a}}\ln\left(x\sqrt{a}+\sqrt{ax^2+b}\right).$$ (a pos.)

70 $$\int \frac{x^2\,dx}{(ax^2+b)^{\frac{3}{2}}} = -\frac{x}{a\sqrt{ax^2+b}} + \frac{1}{a\sqrt{-a}}\sin^{-1}\left(x\sqrt{-\frac{a}{b}}\right).$$ (a neg.)

71 $$\int \frac{dx}{x\,(ax^n+b)} = \frac{1}{bn}\ln\frac{x^n}{ax^n+b}.$$

72 $$\int \frac{dx}{x\sqrt{ax^n+b}} = \frac{1}{n\sqrt{b}}\ln\frac{\sqrt{ax^n+b}-\sqrt{b}}{\sqrt{ax^n+b}+\sqrt{b}}.$$ (b pos.)

73 $$\int \frac{dx}{x\sqrt{ax^n+b}} = \frac{2}{n\sqrt{-b}}\sec^{-1}\sqrt{\frac{-ax^n}{b}}.$$ (b neg.)

Functions containing $ax^2 + bx + c$

74 $$\int \frac{dx}{ax^2+bx+c} = \frac{1}{\sqrt{b^2-4\,ac}}\ln\frac{2\,ax+b-\sqrt{b^2-4\,ac}}{2\,ax+b+\sqrt{b^2-4\,ac}}.$$ ($b^2 > 4\,ac$)

75 $$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4\,ac-b^2}}\tan^{-1}\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}.$$ ($b^2 < 4\,ac$)

76 $$\int \frac{dx}{ax^2+bx+c} = -\frac{2}{2\,ax+b}.$$ ($b^2 = 4\,ac$)

77 $\int \frac{x\,dx}{ax^2+bx+c} = \frac{1}{2a}\ln(ax^2+bx+c) - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}.$

78 $\int \frac{x^2\,dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2}\ln(ax^2+bx+c) + \frac{b^2-2ac}{2a^2}\int\frac{dx}{ax^2+bx+c}.$

79 $\int\frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln\left(2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right).$ (a pos.)

80 $\int\frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{-a}}\sin^{-1}\frac{-2ax-b}{\sqrt{b^2-4ac}}.$ (a neg.)

81 $\int\sqrt{ax^2+bx+c}\,dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$

82 $\int\frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int\frac{dx}{\sqrt{ax^2+bx+c}}.$

83 $\int\sqrt{ax^2+bx+c}\;x\,dx = \frac{(ax^2+bx+c)^{\frac{3}{2}}}{3a} - \frac{b}{2a}\int\sqrt{ax^2+bx+c}\,dx.$

84 $\int\frac{dx}{x\sqrt{ax^2+bx+c}} = -\frac{1}{\sqrt{c}}\ln\left(\frac{\sqrt{ax^2+bx+c}+\sqrt{c}}{x}+\frac{b}{2\sqrt{c}}\right).$ (c pos.)

85 $\int\frac{dx}{x\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{-c}}\sin^{-1}\frac{bx+2c}{x\sqrt{b^2-4ac}}.$ (c neg.)

86 $\int\frac{dx}{x\sqrt{ax^2+bx}} = -\frac{2}{bx}\sqrt{ax^2+bx}.$

87 $\int\frac{dx}{(ax^2+bx+c)^{\frac{3}{2}}} = -\frac{2(2ax+b)}{(b^2-4ac)\sqrt{ax^2+bx+c}}.$

Functions containing sin ax

88 $\int \sin u\,du = -\cos u.$ (u is any function of x)

89 $\int \sin ax\,dx = -\frac{1}{a}\cos ax.$

90 $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}.$

91 $\int \sin^3 ax\,dx = -\frac{1}{a}\cos ax + \frac{1}{3a}\cos^3 ax.$

92 $\int \sin^4 ax\,dx = \frac{3}{8}x - \frac{1}{4a}\sin 2ax + \frac{1}{32a}\sin 4ax.$

93 $\int \sin^n ax\,dx = -\frac{\sin^{n-1} ax\cos ax}{na} + \frac{n-1}{n}\int\sin^{n-2} ax\,dx.$ (n pos. integ.)

94 $\int\frac{dx}{\sin ax} = \frac{1}{a}\ln\tan\frac{ax}{2} = \frac{1}{a}\ln(\csc ax - \cot ax).$

95 $\int\frac{dx}{\sin^2 ax} = -\frac{1}{a}\cot ax.$

96 $\int\frac{dx}{\sin^n ax} = -\frac{1}{a(n-1)}\frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1}\int\frac{dx}{\sin^{n-2} ax}.$ (n integ. > 1)

97 $\int\frac{dx}{1+\sin ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4}-\frac{ax}{2}\right).$

98 $\int\frac{dx}{1-\sin ax} = \frac{1}{a}\cot\left(\frac{\pi}{4}-\frac{ax}{2}\right).$

99 $\int\frac{dx}{b+c\sin ax} = \frac{-2}{a\sqrt{b^2-c^2}}\tan^{-1}\left[\sqrt{\frac{b-c}{b+c}}\tan\left(\frac{\pi}{4}-\frac{ax}{2}\right)\right].$ $(b^2 > c^2)$

100 $\int\frac{dx}{b+c\sin ax} = \frac{-1}{a\sqrt{c^2-b^2}}\ln\frac{c+b\sin ax+\sqrt{c^2-b^2}\cos ax}{b+c\sin ax}.$ $(c^2 > b^2)$

101 $\int\sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}.$ $(a^2 \neq b^2)$

Functions containing cos ax

102 $\int\cos u\,du = \sin u.$ (u is any function of x)

103 $\int\cos ax\,dx = \frac{1}{a}\sin ax.$ $\int\sqrt{1-\cos x}\,dx = \sqrt{2}\int\sin\frac{x}{2}dx.$

104 $\int\cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}.$ $\int\sqrt{1+\cos x}\,dx = \sqrt{2}\int\cos\frac{x}{2}dx.$

105 $\int\cos^3 ax\,dx = \frac{1}{a}\sin ax - \frac{1}{3a}\sin^3 ax.$

106 $\int\cos^4 ax\,dx = \frac{3}{8}x + \frac{1}{4a}\sin 2ax + \frac{1}{32a}\sin 4ax.$

107 $\int\cos^n ax\,dx = \frac{\cos^{n-1} ax\sin ax}{na} + \frac{n-1}{n}\int\cos^{n-2} ax\,dx.$ (n pos. integ.)

108 $\int\frac{dx}{\cos ax} = \frac{1}{a}\ln\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right) = \frac{1}{a}\ln(\tan ax + \sec ax).$

109 $\int\frac{dx}{\cos^2 ax} = \frac{1}{a}\tan ax.$

110 $\int\frac{dx}{\cos^n ax} = \frac{1}{a(n-1)}\frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1}\int\frac{dx}{\cos^{n-2} ax}.$ (n integ. > 1)

111 $\int\frac{dx}{1+\cos ax} = \frac{1}{a}\tan\frac{ax}{2}.$

112 $\int\frac{dx}{1-\cos ax} = -\frac{1}{a}\cot\frac{ax}{2}.$

113 $\int\frac{dx}{b+c\cos ax} = \frac{2}{a\sqrt{b^2-c^2}}\tan^{-1}\left(\sqrt{\frac{b-c}{b+c}}\tan\frac{ax}{2}\right).$ $(b^2 > c^2)$

114 $\int\frac{dx}{b+c\cos ax} = \frac{1}{a\sqrt{c^2-b^2}}\ln\frac{c+b\cos ax+\sqrt{c^2-b^2}\sin ax}{b+c\cos ax}.$ $(c^2 > b^2)$

115 $\int\cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}.$ $(a^2 \neq b^2)$

Functions containing sin ax and cos ax

116 $\int\sin ax\cos bx\,dx = -\frac{1}{2}\left[\frac{\cos(a-b)x}{a-b}+\frac{\cos(a+b)x}{a+b}\right].$ $(a^2 \neq b^2)$

117 $\int\sin^n ax\cos ax\,dx = \frac{1}{a(n+1)}\sin^{n+1} ax.$ $(n \neq -1).$

118 $\int\frac{\cos ax}{\sin ax}dx = \frac{1}{a}\ln\sin ax.$

119 $\int (b + c\sin ax)^n \cos ax\,dx = \frac{1}{ac(n+1)}(b + c\sin ax)^{n+1}$. $(n \neq -1)$

120 $\int \frac{\cos ax\,dx}{b + c\sin ax} = \frac{1}{ac}\ln(b + c\sin ax)$.

121 $\int \cos^n ax \sin ax\,dx = -\frac{1}{a(n+1)}\cos^{n+1} ax$. $(n \neq -1)$.

122 $\int \frac{\sin ax}{\cos ax}dx = -\frac{1}{a}\ln\cos ax$.

123 $\int (b + c\cos ax)^n \sin ax\,dx = -\frac{1}{ac(n+1)}(b + c\cos ax)^{n+1}$. $(n \neq -1)$

124 $\int \frac{\sin ax}{b + c\cos ax}dx = -\frac{1}{ac}\ln(b + c\cos ax)$.

125 $\int \frac{dx}{b\sin ax + c\cos ax} = \frac{1}{a\sqrt{b^2 + c^2}}\ln\left[\tan\frac{1}{2}\left(ax + \tan^{-1}\frac{c}{b}\right)\right]$.

126 $\int \sin^2 ax\cos^2 ax\,dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$.

127 $\int \frac{dx}{\sin ax\cos ax} = \frac{1}{a}\ln\tan ax$.

128 $\int \frac{dx}{\sin^2 ax\cos^2 ax} = \frac{1}{a}(\tan ax - \cot ax)$.

129 $\int \frac{\sin^2 ax}{\cos ax}dx = \frac{1}{a}\left[-\sin ax + \ln\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\right]$.

130 $\int \frac{\cos^2 ax}{\sin ax}dx = \frac{1}{a}\left[\cos ax + \ln\tan\frac{ax}{2}\right]$.

131 $\int \sin^m ax\cos^n ax\,dx = -\frac{\sin^{m-1} ax\cos^{n+1} ax}{a(m+n)}$

$+\frac{m-1}{m+n}\int \sin^{m-2} ax\cos^n ax\,dx$. (m, n pos.)

132 $\int \sin^m ax\cos^n ax\,dx = \frac{\sin^{m+1} ax\cos^{n-1} ax}{a(m+n)}$

$+\frac{n-1}{m+n}\int \sin^m ax\cos^{n-2} ax\,dx$. (m, n pos.)

133 $\int \frac{\cos^n ax}{\sin^m ax}dx = \frac{-\cos^{n+1} ax}{a(m-1)\sin^{m-1} ax}$

$+\frac{m-n-2}{(m-1)}\int \frac{\cos^n ax}{\sin^{m-2} ax}dx$. (m, n pos., $m \neq 1$)

134 $\int \frac{\sin^m ax}{\cos^n ax}dx = \frac{\sin^{m+1} ax}{a(n-1)\cos^{n-1} ax}$

$-\frac{m-n+2}{n-1}\int \frac{\sin^m ax}{\cos^{n-2} ax}dx$. (m, n pos., $n \neq 1$)

135 $\int \frac{\sin^{2n} ax}{\cos ax}dx = \int \frac{(1 - \cos^2 ax)^n}{\cos ax}dx$. (Expand, divide, and use 103–108)

136 $\int \frac{\cos^{2n} ax}{\sin ax}dx = \int \frac{(1 - \sin^2 ax)^n}{\sin ax}dx$. (Expand, divide, and use 89–94)

137 $\int \frac{\sin^{2n+1} ax}{\cos ax}dx = \int \frac{(1 - \cos^2 ax)^n}{\cos ax}\sin ax\,dx$.

(Expand, divide, and use 121–122)

138 $\int \frac{\cos^{2n+1} ax}{\sin ax}dx = \int \frac{(1 - \sin^2 ax)^n}{\sin ax}\cos ax\,dx$.

(Expand, divide, and use 117–118)

Functions containing $\tan ax \left(= \frac{1}{\cot ax}\right)$ or $\cot ax \left(= \frac{1}{\tan ax}\right)$

139 $\int \tan u\,du = -\ln\cos u$. (u is any function of x)

140 $\int \tan ax\,dx = -\frac{1}{a}\ln\cos ax$.

141 $\int \tan^2 ax\,dx = \frac{1}{a}\tan ax - x$.

142 $\int \tan^n ax\,dx = \frac{1}{a(n-1)}\tan^{n-1} ax - \int \tan^{n-2} ax\,dx$. (n integ. > 1)

143 $\int \cot u\,du = \ln\sin u$. (u is any function of x)

144 $\int \cot ax\,dx = \int \frac{dx}{\tan ax} = \frac{1}{a}\ln\sin ax$.

145 $\int \cot^2 ax\,dx = \int \frac{dx}{\tan^2 ax} = -\frac{1}{a}\cot ax - x$.

146 $\int \cot^n ax\,dx = \int \frac{dx}{\tan^n ax} = -\frac{1}{a(n-1)}\cot^{n-1} ax - \int \cot^{n-2} ax\,dx$.

(n integ. > 1)

147 $\int \frac{dx}{b + c\tan ax} = \int \frac{\cot ax\,dx}{b\cot ax + c} = \frac{1}{b^2 + c^2}\left[bx + \frac{c}{a}\ln(b\cos ax + c\sin ax)\right]$.

148 $\int \frac{dx}{b + c\cot ax} = \int \frac{\tan ax\,dx}{b\tan ax + c} = \frac{1}{b^2 + c^2}\left[bx - \frac{c}{a}\ln(c\cos ax + b\sin ax)\right]$.

149 $\int \frac{dx}{\sqrt{1 + \tan^2 ax}} = \frac{1}{a}\sin ax$.

150 $\int \frac{dx}{\sqrt{b + c\tan^2 ax}} = \frac{1}{a\sqrt{b - c}}\sin^{-1}\left(\sqrt{\frac{b - c}{b}}\sin ax\right)$. (b pos., $b^2 > c^2$)

Functions containing $\sec ax \left(= \frac{1}{\cos ax}\right)$ or $\csc ax \left(= \frac{1}{\sin ax}\right)$

151 $\int \sec u\,du = \ln(\sec u + \tan u) = \ln\tan\left(\frac{u}{2} + \frac{\pi}{4}\right)$. (u is any function of x)

152 $\int \sec ax\,dx = \frac{1}{a}\ln\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)$.

153 $\int \sec^2 ax\,dx = \frac{1}{a}\tan ax$.

154 $\int \sec^n ax\,dx = \frac{1}{a(n-1)}\frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1}\int \sec^{n-2} ax\,dx$. (n integ. > 1)

155 $\int \csc u\,du = \ln(\csc u - \cot u) = \ln\tan\frac{u}{2}$. (u is any function of x)

156 $\int \csc ax\,dx = \frac{1}{a}\ln\tan\frac{ax}{2}.$

157 $\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax.$

158 $\int \csc^n ax\,dx = -\frac{1}{a(n-1)}\frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1}\int \csc^{n-2} ax\,dx.$ (n integ. > 1)

Functions containing tan ax and sec ax or cot ax and csc ax

159 $\int \tan u \sec u\,du = \sec u.$ (u is any function of x)

160 $\int \tan ax \sec ax\,dx = \frac{1}{a}\sec ax.$

161 $\int \tan^n ax \sec^2 ax\,dx = \frac{1}{a(n+1)}\tan^{n+1} ax.$ $(n \neq -1)$

162 $\int \frac{\sec^2 ax\,dx}{\tan ax} = \frac{1}{a}\ln\tan ax.$

163 $\int \cot u \csc u\,du = -\csc u.$ (u is any function of x)

164 $\int \cot ax \csc ax\,dx = -\frac{1}{a}\csc ax.$

165 $\int \cot^n ax \csc^2 ax\,dx = -\frac{1}{a(n+1)}\cot^{n+1} ax.$ $(n \neq -1)$

166 $\int \frac{\csc^2 ax\,dx}{\cot ax} = -\frac{1}{a}\ln\cot ax.$

Inverse Trigonometric Functions

167 $\int \sin^{-1} ax\,dx = x\sin^{-1} ax + \frac{1}{a}\sqrt{1-a^2x^2}.$

168 $\int \cos^{-1} ax\,dx = x\cos^{-1} ax - \frac{1}{a}\sqrt{1-a^2x^2}.$

169 $\int \tan^{-1} ax\,dx = x\tan^{-1} ax - \frac{1}{2a}\ln(1+a^2x^2).$

170 $\int \cot^{-1} ax\,dx = x\cot^{-1} ax + \frac{1}{2a}\ln(1+a^2x^2).$

171 $\int \sec^{-1} ax\,dx = x\sec^{-1} ax - \frac{1}{a}\ln\left(ax+\sqrt{a^2x^2-1}\right).$

172 $\int \csc^{-1} ax\,dx = x\csc^{-1} ax + \frac{1}{a}\ln\left(ax+\sqrt{a^2x^2-1}\right).$

Algebraic and Trigonometric Functions

173 $\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax.$

174 $\int x^n \sin ax\,dx = -\frac{1}{a}x^n\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\,dx.$ (n pos.)

175 $\int \frac{\sin ax\,dx}{x} = ax - \frac{(ax)^3}{3\,|\underline{3}} + \frac{(ax)^5}{5\,|\underline{5}} - \cdots.$

176 $\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax.$

177 $\int x^n \cos ax\,dx = \frac{1}{a}x^n\sin ax - \frac{n}{a}\int x^{n-1}\sin ax\,dx.$ (n pos.)

178 $\int \frac{\cos ax\,dx}{x} = \ln ax - \frac{(ax)^2}{2\,|\underline{2}} + \frac{(ax)^4}{4\,|\underline{4}} - \cdots$

Exponential, Algebraic, Trigonometric, Logarithmic Functions

179 $\int b^u\,du = \frac{b^u}{\ln b}.$ (u is any function of x)

180 $\int e^u\,du = e^u.$ (u is any function of x)

181 $\int b^{ax}\,dx = \frac{b^{ax}}{a\ln b}.$

182 $\int e^{ax}\,dx = \frac{1}{a}e^{ax}.$

183 $\int \frac{dx}{b+ce^{ax}} = \frac{1}{ab}\left[ax - \ln(b+ce^{ax})\right].$

184 $\int \frac{e^{ax}\,dx}{b+ce^{ax}} = \frac{1}{ac}\ln(b+ce^{ax}).$

185 $\int \frac{dx}{be^{ax}+ce^{-ax}} = \frac{1}{a\sqrt{bc}}\tan^{-1}\left(e^{ax}\sqrt{\frac{b}{c}}\right).$ (b and c pos.)

186 $\int xb^{ax}\,dx = \frac{xb^{ax}}{a\ln b} - \frac{b^{ax}}{a^2(\ln b)^2}.$

187 $\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1).$

188 $\int x^n b^{ax}\,dx = \frac{x^n b^{ax}}{a\ln b} - \frac{n}{a\ln b}\int x^{n-1}b^{ax}\,dx.$ (n pos.)

189 $\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx.$ (n pos.)

190 $\int \frac{e^{ax}}{x}\,dx = \ln x + ax + \frac{(ax)^2}{2\,|\underline{2}} + \frac{(ax)^3}{3\,|\underline{3}} + \cdots.$

191 $\int \frac{e^{ax}}{x^n}\,dx = \frac{1}{n-1}\left[-\frac{e^{ax}}{x^{n-1}} + a\int \frac{e^{ax}}{x^{n-1}}\,dx\right].$ (n integ. > 1)

192 $\int e^{ax}\ln x\,dx = \frac{1}{a}e^{ax}\ln x - \frac{1}{a}\int \frac{e^{ax}}{x}\,dx.$

193 $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx).$

194 $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx).$

195 $\int xe^{ax}\sin bx\,dx = \frac{xe^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) - \frac{e^{ax}}{(a^2+b^2)^2}\left[(a^2-b^2)\sin bx - 2ab\cos bx\right].$

196 $\int xe^{ax}\cos bx\,dx = \frac{xe^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

$$- \frac{e^{ax}}{(a^2+b^2)^2}[(a^2-b^2)\cos bx + 2\,ab\sin bx].$$

197 $\int \ln ax\,dx = x\ln ax - x.$

198 $\int (\ln ax)^n\,dx = x(\ln ax)^n - n\int(\ln ax)^{n-1}\,dx.$ (n pos.)

199 $\int x^n\ln ax\,dx = x^{n+1}\left[\frac{\ln ax}{n+1} - \frac{1}{(n+1)^2}\right].$ $(n \neq -1)$

200 $\int\frac{(\ln ax)^n}{x}\,dx = \frac{(\ln ax)^{n+1}}{n+1}.$ $(n \neq -1)$

201 $\int\frac{dx}{x\ln ax} = \ln(\ln ax).$

202 $\int\frac{dx}{\ln ax} = \frac{1}{a}\left[\ln(\ln ax) + \ln ax + \frac{(\ln ax)^2}{2\,\underline{|2}} + \frac{(\ln ax)^3}{3\,\underline{|3}} + \cdots\right].$

203 $\int\sin(\ln ax)\,dx = \frac{x}{2}[\sin(\ln ax) - \cos(\ln ax)].$

204 $\int\cos(\ln ax)\,dx = \frac{x}{2}[\sin(\ln ax) + \cos(\ln ax)].$

Some Definite Integrals

205 $\int_0^a\sqrt{a^2-x^2}\,dx = \frac{\pi a^2}{4}.$

206 $\int_0^a\sqrt{2\,ax-x^2}\,dx = \frac{\pi a^2}{4}.$

207 $\int_0^\infty\frac{dx}{ax^2+b} = \frac{\pi}{2\sqrt{ab}}.$ (a and b pos.)

208 $\int_0^{\sqrt{\frac{b}{a}}}\frac{dx}{ax^2+b} = \int_{\sqrt{\frac{b}{a}}}^\infty\frac{dx}{ax^2+b} = \frac{\pi}{4\sqrt{ab}}.$ (a and b pos.)

209 $\int_0^{\frac{\pi}{2}}\sin^n ax\,dx = \int_0^{\frac{\pi}{2}}\cos^n ax\,dx = \frac{1\cdot3\cdot5\cdot\ldots(n-1)}{2\cdot4\cdot6\cdot\ldots n}\,\frac{\pi}{2a}.$ (n, pos. even integ.)

210 $\int_0^{\frac{\pi}{2}}\sin^n ax\,dx = \int_0^{\frac{\pi}{2}}\cos^n ax\,dx = \frac{2\cdot4\cdot6\cdot\ldots(n-1)}{1\cdot3\cdot5\cdot\ldots n}\,\frac{1}{a}.$ (n, pos. odd integ.)

211 $\int_0^\pi\sin ax\sin bx\,dx = \int_0^\pi\cos ax\cos bx\,dx = 0.$ $(a \neq b)$

212 $\int_0^\pi\sin^2 ax\,dx = \int_0^\pi\cos^2 ax\,dx = \frac{\pi}{2}.$

213 $\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}.$

214 $\int_0^\infty x^ne^{-ax}\,dx = \frac{\underline{|n}}{a^{n+1}}.$ (n pos. integ.)

HYPERBOLIC FUNCTIONS

Definitions of Hyperbolic Functions

Hyperbolic sine (sinh) $x = \frac{1}{2}(e^x - e^{-x})$; $\qquad \operatorname{csch} x = \frac{1}{\sinh x}$

Hyperbolic cosine (cosh) $x = \frac{1}{2}(e^x + e^{-x})$; $\qquad \operatorname{sech} x = \frac{1}{\cosh x}$

Hyperbolic tangent (tanh) $x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$; $\qquad \coth x = \frac{1}{\tanh x}$

where e = base of natural logarithms.

Inverse or Anti-Hyperbolic Functions

If $x = \sinh y$, then y is the anti-hyperbolic sine of x or $y = \sinh^{-1} x$.

$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$; $\qquad \operatorname{csch}^{-1} x = \sinh^{-1}\frac{1}{x}.$

$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$; $\qquad \operatorname{sech}^{-1} x = \cosh^{-1}\frac{1}{x}.$

$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x}$; $\qquad \coth^{-1} x = \tanh^{-1}\frac{1}{x}.$

Derivatives of Hyperbolic Functions

$\frac{d}{dx}\sinh x = \cosh x$; $\quad \frac{d}{dx}\cosh x = \sinh x$; $\quad \frac{d}{dx}\tanh x = \operatorname{sech}^2 x.$

$\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$; $\quad \frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x\tanh x$; $\quad \frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x\coth x.$

$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}$; $\quad \frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$; $\quad \frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}.$

$\frac{d}{dx}\coth^{-1} x = -\frac{1}{x^2-1}$; $\quad \frac{d}{dx}\operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}}$; $\quad \frac{d}{dx}\operatorname{csch}^{-1} x = -\frac{1}{x\sqrt{x^2+1}}.$

Some Integrals Leading to Hyperbolic Functions

$\int\sinh x\,dx = \cosh x$; $\quad \int\cosh x\,dx = \sinh x$; $\quad \int\tanh x\,dx = \ln\cosh x.$

$\int\coth x\,dx = \ln\sinh x$; $\quad \int\operatorname{sech} x\,dx = \sin^{-1}(\tanh x)$; $\quad \int\operatorname{csch} x\,dx = \ln\tanh\frac{x}{2}.$

$\int\frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\frac{x}{a}$; $\quad \int\frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\frac{x}{a}$; $\quad \int\frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}\frac{x}{a}.$ $(x < a)$

$\int\frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\sinh^{-1}\frac{a}{x}$; $\quad \int\frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\cosh^{-1}\frac{a}{x}$;

$\int\frac{dx}{x^2-a^2} = -\frac{1}{a}\tanh^{-1}\frac{a}{x}.$ $(x > a)$

$\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}\frac{x}{a}.$

$\int\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\sinh^{-1}\frac{x}{a}.$