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SAUNDERS SHORT TABLES

Mathematical and Physical Tables for Students

Compiled by R. Stevenson
McGill University

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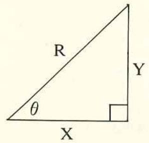
TRIGONOMETRIC FORMULAS



$\sin\theta = \dfrac{Y}{R}$ $\qquad$ $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{X}{R}$ $\qquad$ $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{Y}{X}$ $\qquad$ $\cot\theta = \dfrac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ π radians $= 180°$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

Series expansions

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots \qquad (-\infty < \theta < \infty)$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots \qquad (-\infty < \theta < \infty)$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \frac{17\theta^7}{315} + \cdots \qquad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$$

$$\cot\theta = \frac{1}{\theta} - \frac{\theta}{3} - \frac{\theta^3}{45} + \cdots \qquad (-\pi < \theta < \pi)$$

$$\sin^{-1} x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots \qquad (-1 \leqslant x \leqslant 1)$$

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

Angles in different quadrants

$\theta =$	$-\phi$	$\pi/2 \pm \phi$	$\pi \pm \phi$	$3\pi/2 \pm \phi$	$2\pi \pm \phi$
$\sin\theta =$	$-\sin\phi$	$+\cos\phi$	$\mp\sin\phi$	$-\cos\phi$	$\pm\sin\phi$
$\cos\theta =$	$+\cos\phi$	$\mp\sin\phi$	$-\cos\phi$	$\pm\sin\phi$	$+\cos\phi$
$\tan\theta =$	$-\tan\phi$	$\mp\cot\phi$	$\pm\tan\phi$	$\mp\cot\phi$	$\pm\tan\phi$

Sums and differences

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\sin\theta + \sin\phi = 2\sin\frac{\theta+\phi}{2}\cdot\cos\frac{\theta-\phi}{2}$$

$$\sin\theta - \sin\phi = 2\cos\frac{\theta+\phi}{2}\cdot\sin\frac{\theta-\phi}{2}$$

$$\cos\theta + \cos\phi = 2\cos\frac{\theta+\phi}{2}\cdot\cos\frac{\theta-\phi}{2}$$

$$\cos\theta - \cos\phi = -2\sin\frac{\theta+\phi}{2}\cdot\sin\frac{\theta-\phi}{2}$$

$$\sin^2\theta - \sin^2\phi = \sin(\theta+\phi)\sin(\theta-\phi)$$

$$\cos^2\theta - \cos^2\phi = -\sin(\theta+\phi)\sin(\theta-\phi)$$

$$\cos^2\theta - \sin^2\phi = \cos(\theta+\phi)\cos(\theta-\phi)$$

De Moivre's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

Multiple angles

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

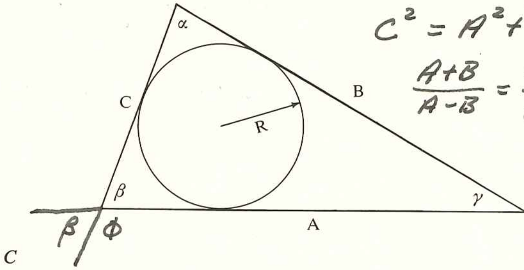
$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

SOLUTION OF TRIANGLES

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

$$\frac{A+B}{A-B} = \frac{\tan\left(\frac{1}{2}(\alpha+\beta)\right)}{\tan\left(\frac{1}{2}(\alpha-\beta)\right)}$$



ϕ = exterior angle
$= 180 - \beta$
$= \alpha + \gamma$

Given A, B, C

$$S = \tfrac{1}{2}(A + B + C)$$

$$R = [S^{-1}(S-A)(S-B)(S-C)]^{1/2}$$

$$\text{area} = RS$$

$$\tan\frac{\alpha}{2} = \frac{R}{S-A} \qquad \tan\frac{\beta}{2} = \frac{R}{S-B} \qquad \tan\frac{\gamma}{2} = \frac{R}{S-C}$$

Given A, B, α

$$\sin\beta = \frac{B}{A}\sin\alpha$$

If $A > B$ then $\beta < \dfrac{\pi}{2}$

if $A < B$, β has two values β_1, $\beta_2 = \pi - \beta_1$

$$\gamma = \pi - (\alpha + \beta)$$

$$C = A\frac{\sin\gamma}{\sin\alpha}$$

$$\text{area} = \tfrac{1}{2}AB\sin\gamma$$

Given A, B, γ

$$\tan\tfrac{1}{2}(\alpha - \beta) = \frac{A-B}{A+B}\cdot\cot\tfrac{1}{2}\gamma$$

$$\alpha + \beta = \pi - \gamma$$

$$C = A\frac{\sin\gamma}{\sin\alpha}$$

$$\text{area} = \tfrac{1}{2}AB\sin\gamma$$

Given A, α, β

$$B = A\frac{\sin\beta}{\sin\alpha}$$

$$C = A\frac{\sin\gamma}{\sin\alpha}$$

$$\gamma = \pi - (\alpha + \beta)$$

$$\text{area} = \tfrac{1}{2}AB\sin\gamma$$

$$\sin\tfrac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\tfrac{1}{2}\theta = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\tfrac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

FORMULAS

Logarithms

$$\log ab = \log a + \log b$$

$$\log a^n = n \log a$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log a^{1/n} = \frac{1}{n} \log a.$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

if $x = a^y$, then $y = \log_a x$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad (-1 < x < 1)$$

$$\ln x = \frac{x-1}{x} + \frac{1}{2}\frac{(x-1)^2}{x^2} + \frac{1}{3}\frac{(x-1)^3}{x^3} + \cdots \qquad (x > \tfrac{1}{2})$$

$$\ln x = 2\left[\frac{x-1}{x+1} + \frac{1}{3}\left(\frac{x-1}{x+1}\right)^3 + \frac{1}{5}\left(\frac{x-1}{x+1}\right)^5 + \cdots\right] \qquad (x > 0)$$

Exponentials

$$a^{x+y} = a^x a^y \qquad a^{-1} = \frac{1}{a} \qquad a^{x/y} = \sqrt[y]{a^x}$$

$$(a^x)^y = a^{xy}$$

$$a^{x-y} = \frac{a^x}{a^y} \qquad a^{1/x} = \sqrt[x]{a} \qquad a^{-x} = \frac{1}{a^x}$$

$$a^x = 1 + x \ln a + \frac{(x \ln a)^2}{2!} + \frac{(x \ln a)^3}{3!} + \cdots \qquad (-\infty < x < \infty)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad (-\infty < x < \infty)$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots \qquad (-\infty < x < \infty)$$

$$e^{1/n} \cong 1 + \frac{2}{2n-1} \qquad n > 1$$

$$e^{-1/n} \cong 1 - \frac{2}{2n+1} \qquad n > 1$$

Hyperbolic functions

$$\sinh \theta = \frac{e^\theta - e^{-\theta}}{2}$$

$$\cosh \theta = \frac{e^\theta + e^{-\theta}}{2}$$

$$\sinh \theta = \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots \qquad (-\infty < \theta < \infty)$$

$$\cosh \theta = 1 + \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots \qquad (-\infty < \theta < \infty)$$

$$\tanh \theta = \theta - \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \cdots \qquad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$$

$$\coth \theta = \frac{1}{\theta} + \frac{\theta}{3} - \frac{\theta^3}{45} + \cdots \qquad (-\pi < \theta < \pi)$$

Factorials

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \cdots \cdot (n-1) \cdot n$$

$$\sqrt{2\pi n}\left(\frac{n}{e}\right)^n < n! < \sqrt{2\pi n}\left(\frac{n}{e}\right)^n\left(1 + \frac{1}{12n-1}\right)$$

$$\ln n! \cong n \ln n - n$$

ALGEBRAIC FORMULAS

Binomial theorem

$$(a \pm b)^n = a^n \pm na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \cdots (\pm 1)^s \frac{n!}{(n-s)!s!}a^{n-s}b^s + \cdots$$

Multinomial theorem

In the expansion of $(x_1 + x_2 + x_3 + \cdots)^N$, the coefficient of the general term

$$x_1^{N_1} x_2^{N_2} x_3^{N_3} \cdots x_i^{N_i} \cdots$$

where $N_1 + N_2 + \cdots = \Sigma N_i = N$ is

$$\frac{N!}{N_1! N_2! \cdots N_i! \cdots} = \frac{N!}{\prod_i N_i!}$$

Quadratic equations

$$ax^2 + bx + c = 0$$

$$x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

Cubic equations

$$ax^3 + bx^2 + cx + d = 0$$

substitute $x = y - \dfrac{b}{3a}$ to obtain

$$y^3 + \alpha y + \beta = 0 \qquad \text{where } \alpha = \frac{3ac - b^2}{3a^2}$$

$$\beta = \frac{2b^3 - 9abc + 27a^2 d}{27a^3}$$

Calculate the quantities

$$A = \sqrt[3]{-\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{\alpha^3}{27}}} \qquad B = \sqrt[3]{-\frac{\beta}{2} - \sqrt{\frac{\beta^2}{4} + \frac{\alpha^3}{27}}}$$

Then the solution is

$$y = A + B, \qquad -\tfrac{1}{2}(A + B) \pm i\frac{\sqrt{3}}{2}(A - B)$$

There will be three cases:

(i) $\dfrac{\beta^2}{4} + \dfrac{\alpha^3}{27} > 0$; one real and two imaginary roots

(ii) $\dfrac{\beta^2}{4} + \dfrac{\alpha^3}{27} = 0$; three real roots with at least two equal

(iii) $\dfrac{\beta^2}{4} + \dfrac{\alpha^3}{27} < 0$; three real, unequal roots

For case (iii) an alternative method of solution is obtained by defining

$$\cos \theta = \frac{-\beta/2}{\sqrt{-\alpha^3/27}}$$

Then the roots are

$$2\sqrt{\frac{-\alpha}{3}}\cos\frac{\theta}{3}, \quad 2\sqrt{\frac{-\alpha}{3}}\cos\left(\frac{\theta}{3} + \frac{2\pi}{3}\right), \quad 2\sqrt{\frac{-\alpha}{3}}\cos\left(\frac{\theta}{3} + \frac{4\pi}{3}\right)$$

DIFFERENTIALS

$d\sin^{-1}u = (1-u^2)^{-1/2}du$

$d\cos^{-1}u = -(1-u^2)^{-1/2}du$

$d\tan^{-1}u = (1+u^2)^{-1}du$

$d\sinh^{-1}u = (u^2+1)^{-1/2}du$

$d\cosh^{-1}u = (u^2-1)^{-1/2}du$

$d\tanh^{-1}u = (1-u^2)^{-1}du$

$d(u+v) = du + dv$

$$d(a\,u) = a\,du$$

$$d(u\,v) = v\,du + u\,dv$$

$$d\left(\frac{u}{v}\right) = \frac{1}{v^2}(v\,du - u\,dv)$$

$$d(u^n) = n\,u^{n-1}\,du$$

$$d(\ln u) = \frac{1}{u}\,du$$

$$d(e^u) = e^u\,du$$

$d(e^{au}) = a e^{au}du$

$$d(u^v) = v\,u^{v-1}\,du + u^v \ln u\,dv$$

$d(a^u) = a^u \ln a\, du$

$$d\sin u = \cos u\,du \qquad d\cos u = -\sin u\,du$$

$$d\tan u = \sec^2 u\,du \qquad d\cot u = -\operatorname{cosec}^2 u\,du$$

$$d\sec u = \sec u \tan u\,du \qquad d\operatorname{cosec} u = -\operatorname{cosec} u \cot u\,du$$

$$d\sinh u = \cosh u\,du \qquad d\cosh u = \sinh u\,du$$

$$d\tanh u = \operatorname{sech}^2 u\,du \qquad d\coth u = -\operatorname{cosech}^2 u\,du$$

INDEFINITE INTEGRALS

$\int f(y)dx = \int \frac{f(y)}{y'}dy$

$\int \frac{f'(x)}{f(x)}dx = \log f(x)$

$\int \frac{f'(x)}{2\sqrt{f(x)}}dx = \sqrt{f(x)}$

$$\int u^n\,du = \frac{1}{n+1}u^{n+1}$$

$\int u\,dv = uv - \int v\,du$

$$\int \frac{du}{u} = \ln u$$

$\int \ln u\,du = u\ln u - u$

$$\int a^u\,du = \frac{a^u}{\ln a}$$

$\int a^{bu}du = \frac{a^{bu}}{b\ln a}$

$$\int e^u\,du = e^u$$

$\int e^{au}du = \frac{1}{a}e^{au}$

$$\int u^m e^{au}\,du = \frac{u^m e^{au}}{a} - \frac{m}{a}\int u^{m-1}e^{au}\,du \qquad (n > 0)$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} = -\frac{1}{a}\cot^{-1}\frac{u}{a}$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left(\frac{u-a}{u+a}\right) = -\frac{1}{a}\coth^{-1}\frac{u}{a} \qquad (u^2 > a^2)$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left(\frac{a+u}{a-u}\right) = +\frac{1}{a}\tanh^{-1}\frac{u}{a} \qquad (u^2 < a^2)$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} = -\cos^{-1}\frac{u}{a}$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right)$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\frac{u}{a}$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\frac{u}{a}$$

$$\int \cos u\,du = \sin u$$

$$\int \sin u\,du = -\cos u$$

$$\int \sec^2 u\,du = \tan u$$

$$\int \operatorname{cosec}^2 u\,du = -\cot u$$

$$\int \tan u\,du = -\ln\cos u$$

$$\int \cot u\,du = \ln\sin u$$

$$\int \sinh u\,du = \cosh u$$

$$\int \cosh u\,du = \sinh u$$

$$\int \tanh u\,du = \ln\cosh u$$

$$\int \coth u\,du = \ln\sinh u$$

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du$$

$$\int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du$$

$\int \sin nu\,du = -\frac{1}{n}\cos nu$

$\int \cos nu\,du = \frac{1}{n}\sin nu$

$\int u\sin nu\,du = \frac{\sin nu}{n^2} - \frac{u\cos nu}{n}$

$\int u\cos nu\,du = \frac{\cos nu}{n^2} + \frac{u\sin nu}{n}$

$\int u^2\sin nu\,du = \frac{2u\sin nu}{n^2} - \left(u^2 - \frac{2}{n^2}\right)\frac{\cos nu}{n}$

$\int u^2\cos nu\,du = \frac{2u\cos nu}{n^2} + \left(u^2 - \frac{2}{n^2}\right)\frac{\sin nu}{n}$

$\int \sin u\cos u\,du = \frac{1}{2}\sin^2 x$

$\int \sin mu\sin nu\,du = \frac{\sin(m-n)u}{2(m-n)} - \frac{\sin(m+n)u}{2(m+n)}$

$\int \cos mu\cos nu\,du = \frac{\sin(m-n)u}{2(m-n)} + \frac{\sin(m+n)u}{2(m+n)}$

$\int \sin mu\cos nu\,du = -\frac{\cos(m-n)u}{2(m-n)} - \frac{\cos(m+n)u}{2(m+n)}$

$\int u e^{au}du = e^{au}\left(\frac{u}{a} - \frac{1}{a^2}\right)$

$\int u^2 e^{au}du = e^{au}\left(\frac{u^2}{a} - \frac{2u}{a^2} + \frac{2}{a^3}\right)$

$\int u^3 e^{au}du = e^{au}\left(\frac{u^3}{a} - \frac{3u^2}{a^2} + \frac{6u}{a^3} - \frac{6}{a^4}\right)$

$\int \sin^2 nu\,du = \frac{u}{2} - \frac{1}{4n}\sin 2nu$

$\int \cos^2 nu\,du = \frac{u}{2} + \frac{1}{4n}\sin 2nu$

DEFINITE INTEGRALS

$$\int_0^{\pi/2} \sin^n u\,du = \int_0^{\pi/2} \cos^n u\,du$$

$$= \frac{1\cdot 3\cdot 5\cdot \cdots \cdot (n-1)}{2\cdot 4\cdot 6\cdot \cdots \cdot n}\cdot\frac{\pi}{2} \qquad (n \text{ even})$$

$$= \frac{2\cdot 4\cdot 6\cdot \cdots \cdot (n-1)}{1\cdot 3\cdot 5\cdot \cdots \cdot n} \qquad (n \text{ odd})$$

$$\int_0^{2\pi} \sin u \, du = \int_0^{2\pi} \cos u \, du = 0$$

$$\int_0^{2\pi} \sin^2 u \, du = \int_0^{2\pi} \cos^2 u \, du = \pi$$

$$\int_0^{2\pi} \sin u \cos u \, du = 0$$

$$\int_0^{\pi} \sin^2 m u \, du = \int_0^{\pi} \cos^2 m u \, du = \frac{\pi}{2}$$

$$\int_0^{\pi} \sin k u \cdot \sin m u \, du = \int_0^{\pi} \cos k u \cos m u \, du = 0 \qquad k \neq m$$

$$\int_0^{\infty} \frac{\sin m u \, du}{u} = \frac{\pi}{2} \qquad m > 0$$
$$= 0 \qquad m = 0$$
$$= -\frac{\pi}{2} \qquad m < 0$$

$$\int_0^{\infty} \frac{\cos u \, du}{u} = \infty$$

$$\int_0^{\infty} \frac{\tan u \, du}{u} = \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{a \, du}{a^2 + u^2} = \frac{\pi}{2} \qquad a > 0$$
$$= 0 \qquad a = 0$$
$$= -\frac{\pi}{2} \qquad a < 0$$

$$\int_{-\infty}^{\infty} x^n e^{-\lambda x^2} dx = \begin{cases} 2I_n, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$$\int_0^{\infty} \frac{\sin^2 u}{u^2} \, du = \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{\cos m u \, du}{1 + u^2} = \frac{\pi}{2} e^{-m} \qquad m > 0$$
$$= \frac{\pi}{2} e^{m} \qquad m < 0$$

$$I_n = \int_0^{\infty} x^n e^{-\lambda x^2} dx$$

n	I_n
0	$\frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$
1	$\frac{1}{2\lambda}$
2	$\frac{1}{4}\sqrt{\pi}(\lambda)^{-3/2}$
3	$\frac{1}{2\lambda^2}$
4	$\frac{3}{8}\sqrt{\pi}(\lambda)^{-5/2}$
5	$\frac{1}{\lambda^3}$

$$\int_0^{\infty} e^{-ax} dx = \frac{1}{a} \qquad a > 0$$

$$\int_0^{\infty} e^{-a^2 u^2} du = \frac{\sqrt{\pi}}{2a}$$

$$\int_0^{\infty} u e^{-u^2} du = \frac{1}{2}$$

$$\int_0^{\infty} u^2 e^{-u^2} du = \frac{\sqrt{\pi}}{4}$$

$$\int_0^{\infty} u^n e^{-au} du = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}} & n > -1 \quad a > 0 \\ \frac{n!}{a^{n+1}} & n = +\text{Int} \end{cases}$$

$$\int_0^{\infty} e^{-au} \cos m u \, du = \frac{a}{a^2 + m^2} \qquad a > 0$$

$$\int_0^{\infty} e^{-au} \sin m u \, du = \frac{m}{a^2 + m^2} \qquad a > 0$$

Spherical Harmonics $Y_l^{m_l}(\theta, \phi)$

l	m_l	$r^l Y_l^{m_l}$
0	0	$\sqrt{1/4\pi}$
1	0	$\sqrt{3/4\pi}\, z$
	± 1	$\mp \sqrt{3/8\pi}\,(x \pm iy)$
2	0	$\sqrt{5/16\pi}\,(3z^2 - r^2)$
	± 1	$\mp \sqrt{15/8\pi}\, z(x \pm iy)$
	± 2	$\sqrt{15/32\pi}\,(x \pm iy)^2$
3	0	$\sqrt{7/16\pi}\, z(5z^2 - 3r^2)$
	± 1	$\mp \sqrt{21/64\pi}\,(5z^2 - r^2)(x \pm iy)$
	± 2	$\sqrt{105/32\pi}\, z(x \pm iy)^2$
	± 3	$\mp \sqrt{35/64\pi}\,(x \pm iy)^3$
4	0	$3/8 \sqrt{1/4\pi}\,(35z^4 - 30z^2r^2 + 3r^4)$
	± 1	$\mp 3/4 \sqrt{5/4\pi}\, z(7z^2 - 3r^2)(x \pm iy)$
	± 2	$3/4 \sqrt{5/8\pi}\,(7z^2 - r^2)(x \pm iy)^2$
	± 3	$\mp 3/4 \sqrt{35/4\pi}\, z(x \pm iy)^3$
	± 4	$3/8 \sqrt{35/8\pi}\,(x \pm iy)^4$

Legendre Polynomials

l	m_l	$P_l^{m_l}(x)$	$P_l^{m_l}(\cos\theta)$
0	0	1	1
1	0	x	$\cos\theta$
	1	$(1 - x^2)^{1/2}$	$\sin\theta$
2	0	$\frac{1}{2}(3x^2 - 1)$	$\frac{1}{2}(3\cos^2\theta - 1)$
	1	$3x(1 - x^2)^{1/2}$	$3\cos\theta\sin\theta$
	2	$3(1 - x^2)$	$3\sin^2\theta$
3	0	$\frac{1}{2}x(5x^2 - 3)$	$\frac{1}{2}\cos\theta\,(5\cos^2\theta - 3)$
	1	$\frac{3}{2}(5x^2 - 1)(1 - x^2)^{1/2}$	$\frac{3}{2}\sin\theta\,(5\cos^2\theta - 1)$
	2	$15x(1 - x^2)$	$15\cos\theta\sin^2\theta$
	3	$15(1 - x^2)^{3/2}$	$15\sin^3\theta$
4	0	$\frac{1}{8}(35x^4 - 30x^2 + 3)$	$\frac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3)$
	1	$\frac{5}{2}x(7x^2 - 3)(1 - x^2)^{1/2}$	$\frac{5}{2}\cos\theta\sin\theta\,(7\cos^2\theta - 3)$
	2	$\frac{15}{2}(7x^2 - 1)(1 - x^2)$	$\frac{15}{2}\sin^2\theta\,(7\cos^2\theta - 1)$
	3	$105x(1 - x^2)^{3/2}$	$105\cos\theta\sin^3\theta$
	4	$105(1 - x^2)^2$	$105\sin^4\theta$

π = 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37511
2π = 6.28318 53071 79586 47692 52867 66559 00576 83943 38798 57022
π^2 = 9.86960 44010 89358 61883 44909 99876 15113 53136 99407 24079
$\frac{1}{\pi}$ = 0.31830 98861 83790 67153 77675 26745 02872 40689 19291 48091

TABLE 30. Physical Constants

Quantity	Symbol	MKS	CGS
Electron charge	e	1.6021×10^{-19} C	4.8030×10^{-10} statcoul
Electron rest mass	m_e	9.1091×10^{-31} kg	9.1091×10^{-28} g
Specific charge of electron	e/m_e	1.7588×10^{11} C/kg	5.2727×10^{17} statcoul/g
Proton rest mass	m_p	1.6725×10^{-27} kg	1.6725×10^{-24} g
Neutron rest mass	m_n	1.6748×10^{-27} kg	1.6748×10^{-24} g
Atomic mass unit	u	1.6604×10^{-27} kg	1.6604×10^{-24} g
Velocity of light in vacuum	c	2.9979×10^{8} m/sec	2.9979×10^{10} cm/sec
Gravitational constant	k_G, G	6.670×10^{-11} Nm²/kg²	6.670×10^{-8} dyne cm² g⁻²
Faraday	F	9.6487×10^{4} C/mole	2.8926×10^{14} statcoul/mole
Avogadro's number (molecules/mole)	N_A	6.0225×10^{26} kg mole^{-1}	6.0225×10^{23} mole^{-1}
Molar volume (gram mole)	V_o	2.2414×10^{-2} m³	2.2414×10^{4} cm³
Gas constant per gram mole	R	8.3143 J/°K. gm mole	8.3143×10^{7} erg/°K. mole
Boltzmann's constant	k	1.3805×10^{-23} J/°K.	1.3805×10^{-16} erg/°K.
Planck's constant	h	6.6257×10^{-34} J sec	6.6257×10^{-27} erg sec
Bohr magneton	μ_B	9.2733×10^{-24} J/T	9.2733×10^{-21} erg/G
Nuclear magneton	μ_N	5.0505×10^{-27} J/T	5.0505×10^{-24} erg/G
Quantum of magnetic flux	$\phi = hc/e$	4.1356×10^{-11} Wb	4.1356×10^{-7} G cm²
Electron radius	$r_e = e^2/m_e c^2$	2.8178×10^{-15} m	2.8178×10^{-13} cm
Bohr radius	a_0	5.2917×10^{-11} m	5.2917×10^{-9} cm
Compton wavelength of electron	$\lambda_{ce} = h/m_e c$	2.4262×10^{-12} m	2.4262×10^{-12} cm
Compton wavelength of proton	λ_{cp}	2.1031×10^{-16} m	2.1031×10^{-16} m
Fine structure constant	α	7.2972×10^{-3}	7.2972×10^{-3}
Rydberg constant	R_∞	1.0974×10^{7} m^{-1}	1.0974×10^{5} cm^{-1}

10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milla	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Alpha	Α	α
Beta	Β	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	Ε	ε
Zeta	Ζ	ζ
Eta	Η	η
Theta	Θ	θ
Iota	Ι	ι
Kappa	Κ	κ
Lambda	Λ	λ
Mu	Μ	μ
Nu	Ν	ν
Xi	Ξ	ξ
Omicron	Ο	ο
Pi	Π	π
Rho	Ρ	ρ
Sigma	Σ	σ
Tau	Τ	τ
Upsilon	Υ	υ
Phi	Φ	φ
Chi	Χ	χ
Psi	Ψ	ψ
Omega	Ω	ω