Unit 21 – Session 2

A quick summary of what we've learned so far (Unit 20 and Unit 21):

Force

Electrical		**Gravitational**
$\vec{F}^{elec}_{\substack{\text{on } A \\ \text{by } B}} = \dfrac{kq_A q_B}{r^2}\hat{r}$ $= q_A \vec{E}_B$	$\Rightarrow$	$\vec{F}^{grav}_{\substack{\text{on } A \\ \text{by } B}} = \dfrac{-Gm_A m_B}{r^2}\hat{r}$ $= m_A \vec{Y}_B$
		$\left(\vec{F}^{grav}_{\substack{\text{on } A \\ \text{by Earth}}} = m_A \vec{g}\right)$

<u>Gauss' Law</u>

Electrical		Gravitational
$\lvert\vec{E}\rvert\lvert\vec{A}\rvert\cos\phi = \dfrac{q^{enclosed}}{\varepsilon_0}$ $= 4\pi k q^{enclosed}$	$\Rightarrow$	$\lvert\vec{Y}\rvert\lvert\vec{A}\rvert\cos\phi = -\dfrac{m^{enclosed}}{\gamma_0}$ $= -4\pi G m^{enclosed}$

Instead of forces, an alternative way we approached problems was to use the concepts of work and energy. Recall the general energy equation we developed last semester, and added to earlier this semester.

$$\underbrace{\Delta K + \Delta E^{\text{int}}}_{\text{system energy}} = \underbrace{Q + \sum W^{\,outside} + T^{\,matter} + T^{\,waves} + T^{\,light}}_{\text{energy transfer mechanisms (in/out of system)}}$$

We aren't going to do any problems that involve the last three energy transfer mechanisms.

$$T^{\,matter} = T^{\,waves} = T^{\,light} = 0$$

Also, internal energy changes, ΔE^{int}, and the heat process, Q, typically are only important in Thermodynamics.

$$\Delta E^{\text{int}} = 0 \qquad Q = 0$$

This will really simplify the energy conservation equation, leaving us with:

$$\Delta K = \sum W^{outside}$$

Writing it out with a little more detail:

$$\Delta K = W^{grav} + W^{spring} + W^{elec} + W^{norm} + W^{tens} + \cdots$$

where $W \equiv \left|\vec{F}\right|\left|\Delta \vec{s}\right| \cos\theta$

Activity 21.5.1.

For part **a.**, we moved the cart along the incline from point a to point c. There were 3 forces acting on the cart (our system), so the energy conservation equation can be written:

$$\Delta K = W^{grav} + W^{you} + W^{norm}$$

However, $W^{norm} \equiv \left|\vec{F}^{norm}\right|\left|\Delta \vec{s}\right|\cos 90° = 0\ \mathrm{N\cdot m}$. Also, we pulled at a constant speed, so $\Delta K = 0\ \mathrm{J}$. Therefore:

$$0\ \mathrm{J} = W^{grav} + W^{you} + 0\ \mathrm{N\cdot m}$$

When you did your calculations, you found:

$$W^{grav} = -W^{you} = -\left(2.5\,\mathrm{N}\right)\left(0.3968\,\mathrm{m}\right)\cos 0° = -0.99\,\mathrm{N\cdot m}$$

For part **b.**, we moved the cart straight up from point a to point d, then sideways from point d to point c. Now, there were only 2 forces acting on our system, so the energy equation can be written:

$$\Delta K = \underbrace{\left(W^{grav} + W^{you}\right)}_{a \text{ to } d} + \underbrace{\left(W^{grav} + W^{you}\right)}_{d \text{ to } c}$$

Again, we pulled at a constant speed, so $\Delta K = 0\text{ J}$. Also, $\theta = 90°$ for both forces on the d to c path, so $W = 0\text{ N}\cdot\text{m}$ for the d to c path. Therefore:

$$0\text{ J} = \underbrace{\left(W^{grav} + W^{you}\right)}_{a \text{ to } d} + \underbrace{\left(0\text{ N}\cdot\text{m}\right)}_{d \text{ to } c}$$

When you did your calculations, you found:

$$W^{grav} = -W^{you} = -\left(5.0\text{ N}\right)\left(0.1984\text{ m}\right)\cos 0° = -0.99\text{ N}\cdot\text{m}$$

Activity 21.5.1. (continued)

We ended up with the same answer for part **b.** as we did for part **a.**, even though the two paths from *a* to *c* were different.

For most forces, this is **not** the case (the work done by the friction force is one example).

The purpose of this activity was to remind us that the gravitational force is a conservative force. The work done by a conservative force doesn't depend on the path, it only depends on the starting position (configuration) and the ending position (configuration) of the system.

Activity 21.6.1

Before using our findings from Activity 21.5.1, we are going to continue to practice finding work, but now for an electrical force. We really like writing the force in terms of the electric field (created by other charges).

For all three cases, both the electric field, $\vec{E}$, and the angle, θ, are constant for the whole path, so we can use:

$$\begin{aligned} W^{elec} &= \left|\vec{F}^{elec}\right|\left|\Delta\vec{s}\right|\cos\theta \\ &= q_t\left|\vec{E}\right|\left|\Delta\vec{s}\right|\cos\theta \end{aligned}$$

(no scary integral needed)

Unless otherwise stated, it is usually assumed that a test charge, q_t, is a positive type of charge.

$$\begin{aligned} \cos 0^\circ &= 1 \\ \cos 45^\circ &= 0.71 \\ \cos 90^\circ &= 0 \\ \cos 135^\circ &= -0.71 \\ \cos 180^\circ &= -1 \end{aligned}$$

Activity 21.7.

If a force is a conservative force, then the work done by the force can instead be written as a change in potential energy, $W = -\Delta U$.

An important thing to make note of:

- To use potential energy, the system boundary needs to be expanded to include the source of the conservative force doing the work. For example, to use ΔU^{grav} on Activity 21.5.1, the system would be the cart **and** the Earth.

From last semester, we found that the gravitational force, $\vec{F}^{grav}$, and the spring force, $\vec{F}^{spring}$, are both conservative forces. Since the **gravitational** force and the **electrical** force, $\vec{F}^{elec}$, are so, so, so similar, we can expect that the electric force is also a conservative force, and indeed it is.

Activity 21.7. (continued)

Going back to the energy equation:

$$\Delta K = W^{grav} + W^{spring} + W^{elec} + W^{norm} + W^{tens} + \cdots$$

We can now rewrite it as:

$$\Delta K = \left(-\Delta U^{grav}\right) + \left(-\Delta U^{spring}\right) + \left(-\Delta U^{elec}\right) + \sum W^{outside}_{non-cons}$$

Moving the potential energy terms to the right-hand side so all the energy terms are together (and so we don't have to remember the negative signs):

$$\Delta K + \Delta U^{grav} + \Delta U^{spring} + \Delta U^{elec} = \sum W^{outside}_{non-cons}$$

We will be focusing on the electric potential energy, ΔU^{elec}.

Actually, instead of electric potential energy, scientists find it more useful to introduce a related concept called **electric potential**. (note: the names are unfortunately very similar – not much we can do but be sure to pay attention to which one we are talking about).

The idea behind electric potential is similar to the connection between electric force and electric field, but applied to energy instead of force.

Electric Force and Electric Field

A collection of source charges creates an electric field, $\vec{E}$, everywhere around them. If another charge, q_t, is put into the electric field, then it will experience an electric force given by $\vec{F}^{elec} = q_t\vec{E}$. This allows us to define the electric field as $\vec{E} = \vec{F}^{elec}/q_t$, the force per unit charge (with units of N/C).

Electric Potential Energy and Electric Potential

A collection of source charges creates an electric potential, V, everywhere around them. If another charge, q_t, is put into the electric potential, then the system will have electric potential energy given by $U^{elec} = q_t V$. This allows us to define the electric potential as $V = U^{elec} / q_t$, the potential energy per unit charge (with units of J/C, or volts).

We found last (and this) semester that *changes* in energy are more meaningful to us, so if we move the charge, q_t, around in the electric potential from point a to point b, then the *change* in electric potential energy is $\Delta U^{elec} = q_t \Delta V$.

Activity 21.7.1

- Change this to “Write the equation for the potential difference, ΔV, as a function of $\vec{E}$, $\Delta\vec{s}$, and θ.
- Remember that $W^{elec} = q_t \left|\vec{E}\right| \left|\Delta\vec{s}\right| \cos\theta$.

Activity 21.8

Read this, but we aren't going to do Activity 21.8.1. We will just use the result of the activity:

- A point charge, q, creates an electric potential everywhere around it (all the way out to infinity) given by $V = \frac{kq}{r}$ (where k is Coulomb's constant).

A note from your textbook, page 737

The symbol V is widely used to represent the *volume* of an object, and now we're introducing the same symbol to mean *electric potential*. To make matters more confusing, V is the abbreviation for volts (the units for electric potential, e.g. $V = 120$ V). In printed text, V for electric potential is italicized, while V for volts is not, but you can't make such a distinction in handwritten work. This is not a pleasant state of affairs, but these are commonly accepted symbols. You must be especially alert to the context in which a symbol is used.)

Activity 21.9.1

For a continuous charge distribution, we'll do what we did before – divide the distribution into small segments (the more the better), and treat each segment as a point charge, with all the charge at the center of the segment.

The spreadsheet is on the class resources page, and is already completed.

- Click on cell E9 to see how the Excel formula matches the equation for the electric potential created by a point charge.
- Click on cell B2 to see how we calculated the circumference of the ring.
- Click on cell B9 and C9 to see how we found how much charge was on each segment.

Activity 21.9.2

Skip parts **a.** and **b.**. Doing the integral in part **c.** gives the exact expression for the electric potential created by a ring of charge as:

$$V = \frac{k\,q^{tot}}{\sqrt{a^2 + x^2}}$$

Use this result to do part **d.**.

- Note that this result is only valid for positions on the *x*-axis, passing through the center of the ring. We lose the symmetry of the problem for positions not on the *x*-axis – it is a much, much, much harder problem.

Activity 21.9.3

Skip this activity. I can go through part **a.** individually with anyone who would like to.

Activity 21.10.1

Do your best to do parts **a.** through **d.** before looking at the images on the next pages.

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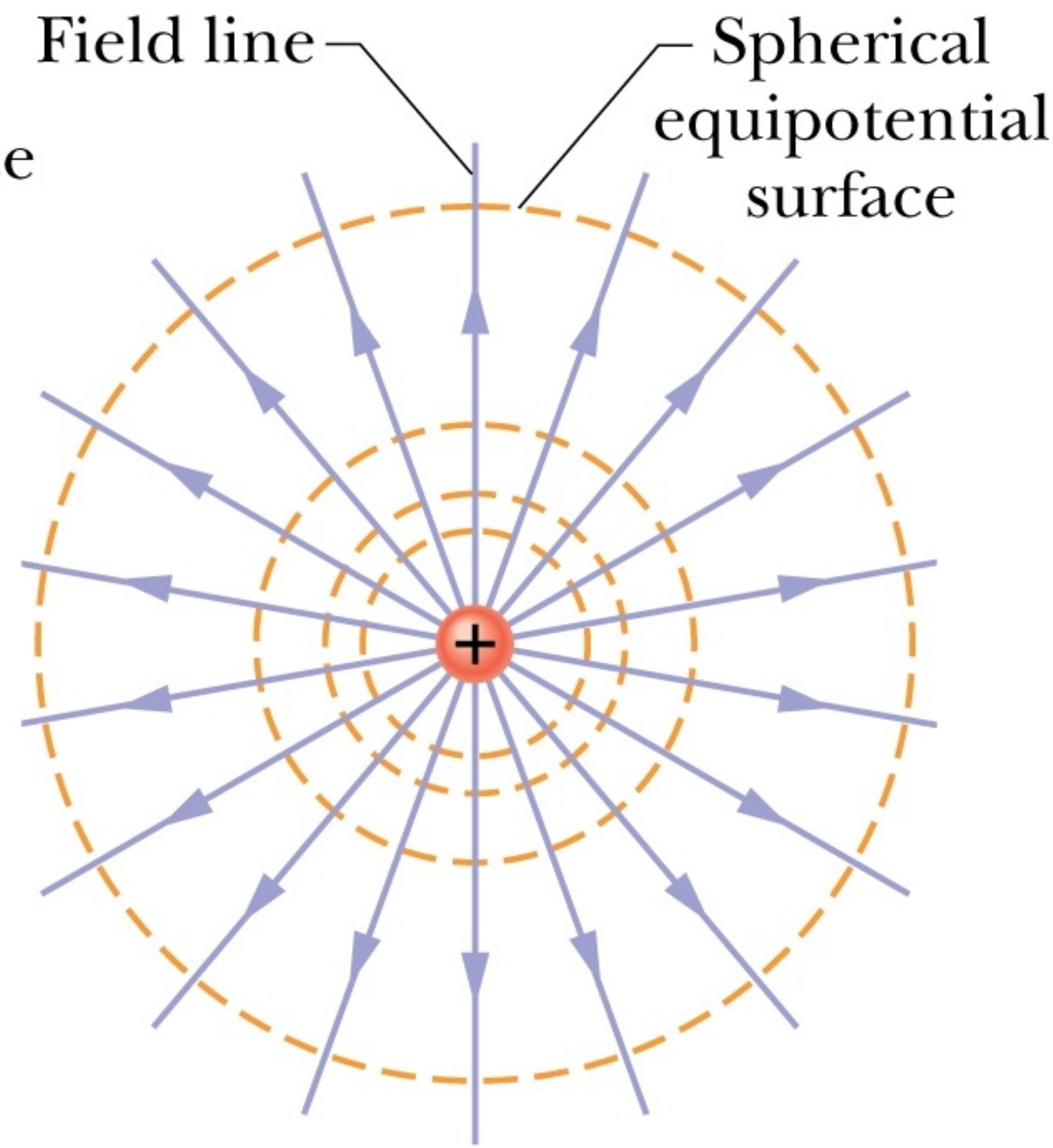
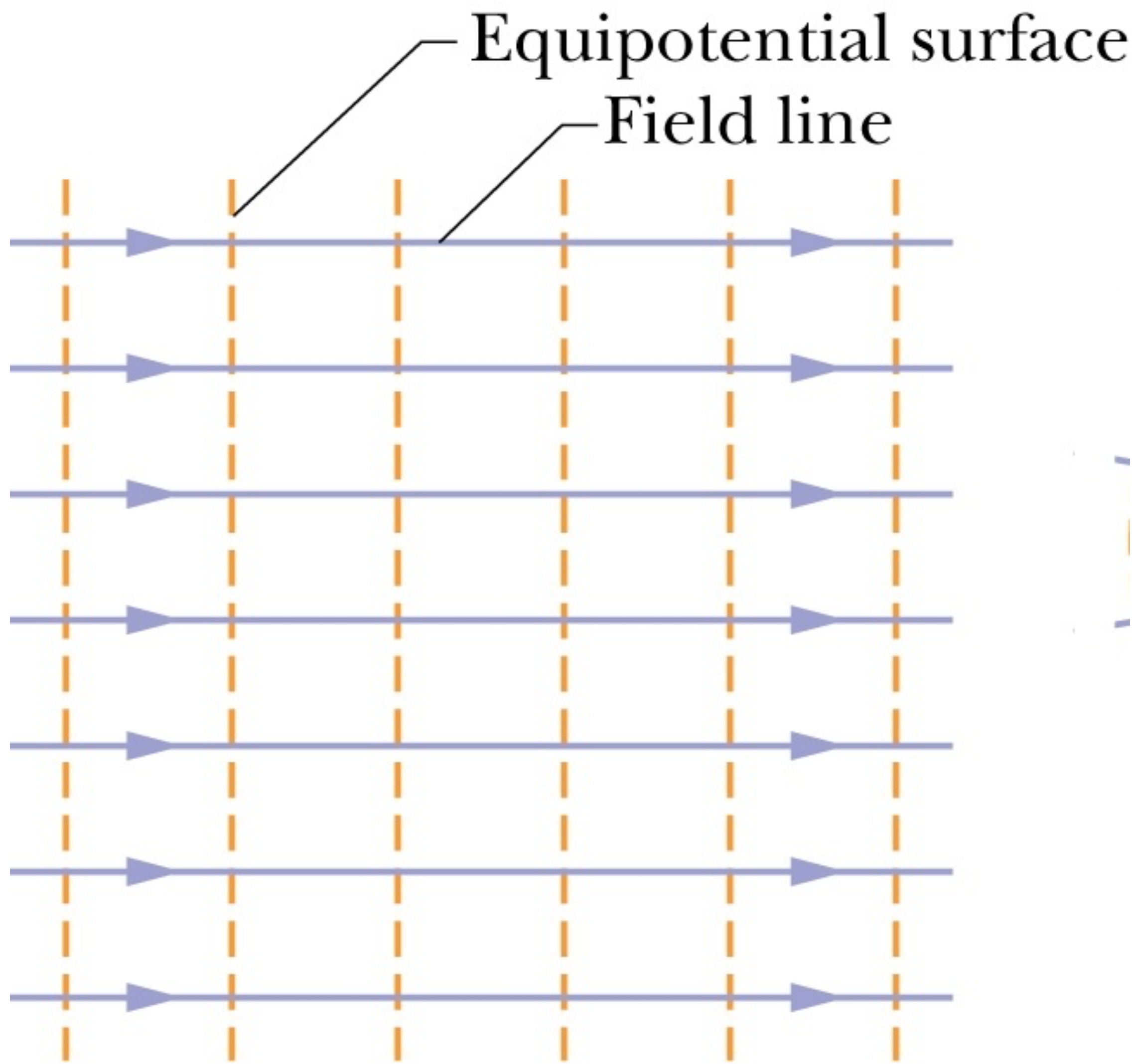
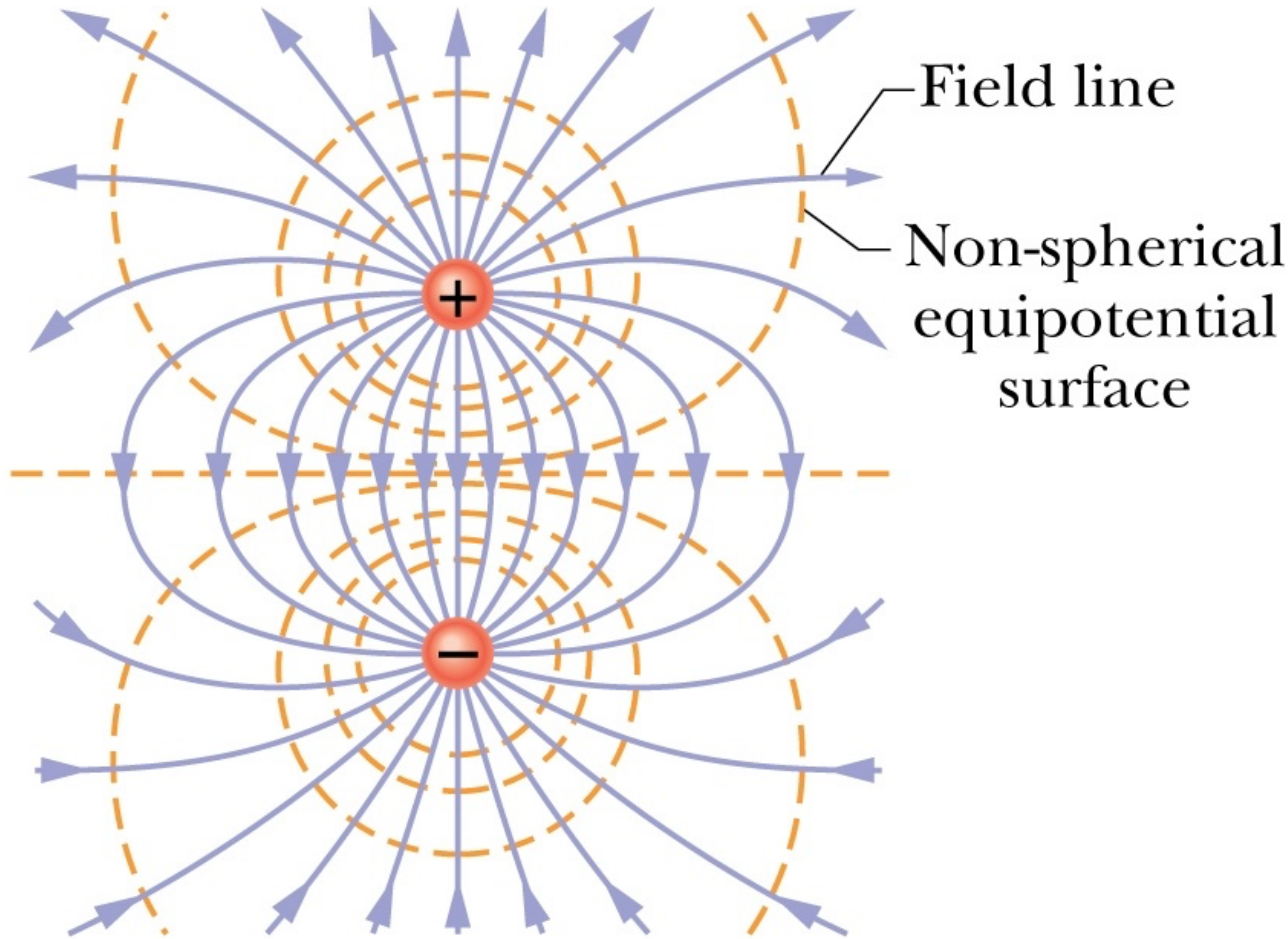



Equipotential surface
Field line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Field line
Non-spherical
equipotential
surface