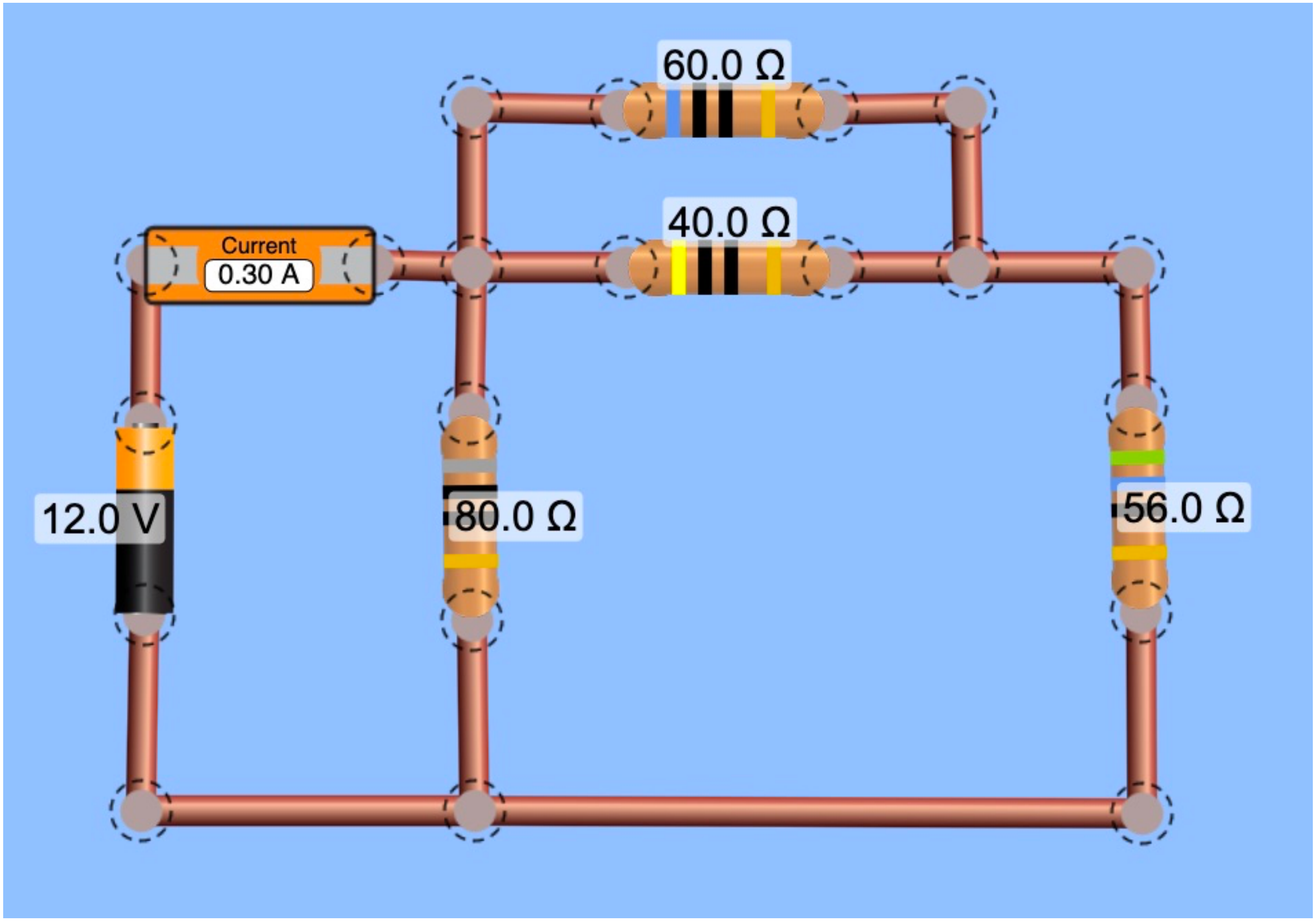
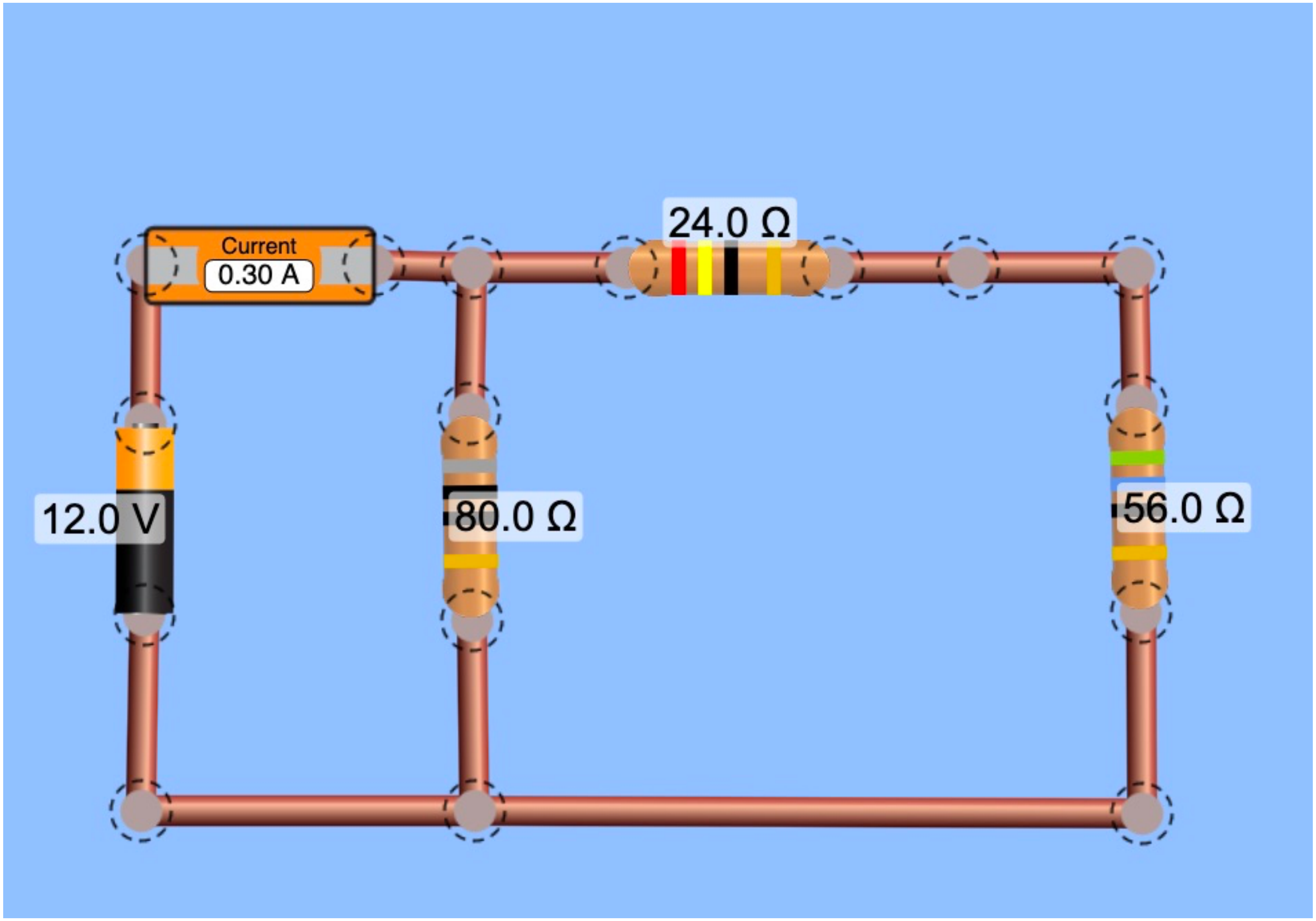
Unit 23 – Session 3

We found in session 2 how to take resistors in series and parallel combinations and find an equivalent resistance.

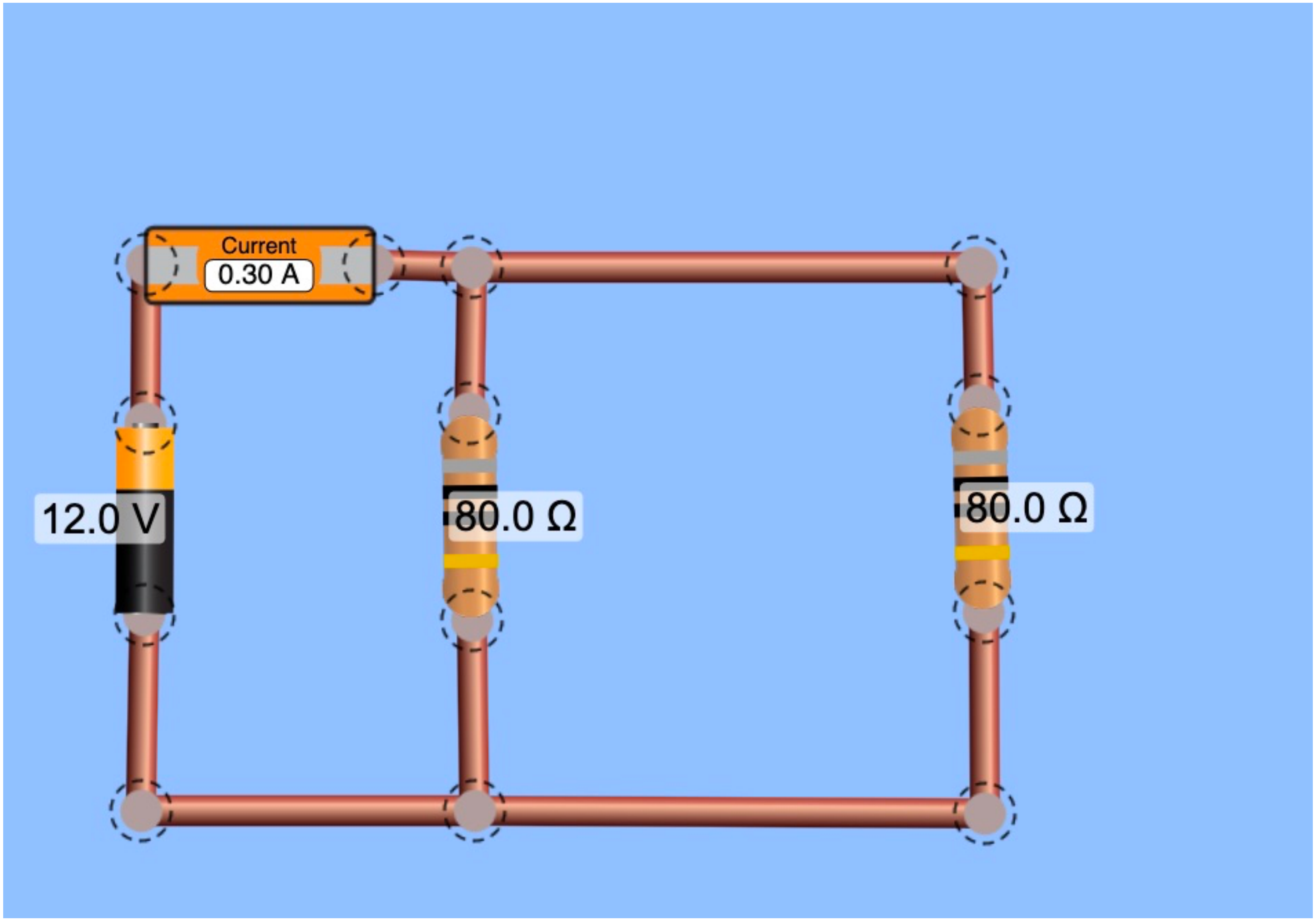
Series: $R_{eq} = R_1 + R_2 + R_3 + \cdots$ (greater than largest R_n)

Parallel: $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \right)^{-1}$ (less than smallest R_n)

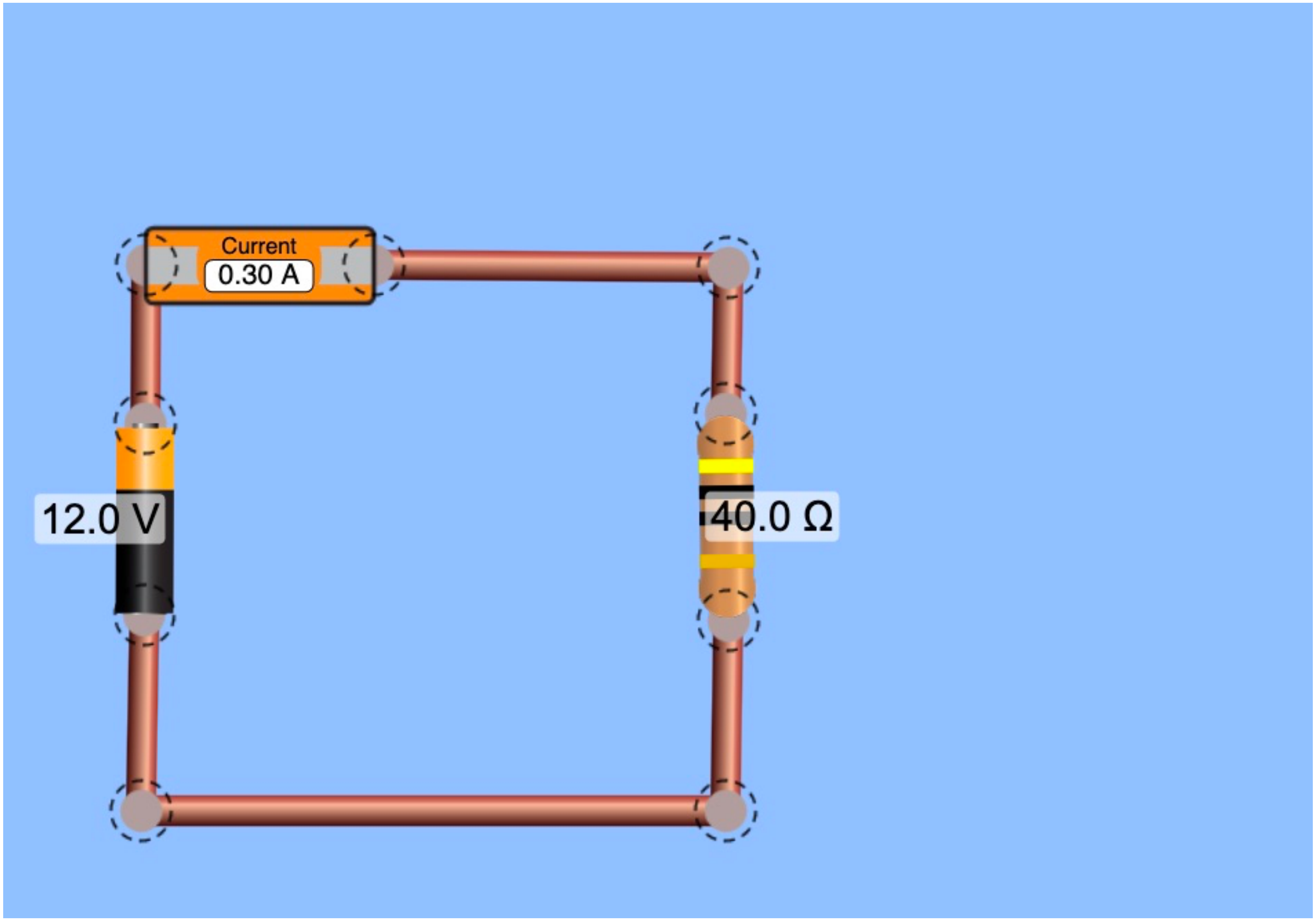

60.0 Ω
Current
0.30 A
40.0 Ω
12.0 V
80.0 Ω
56.0 Ω



Current
0.30 A
24.0 Ω
12.0 V
80.0 Ω
56.0 Ω



Current
0.30 A
12.0 V
80.0 Ω
80.0 Ω

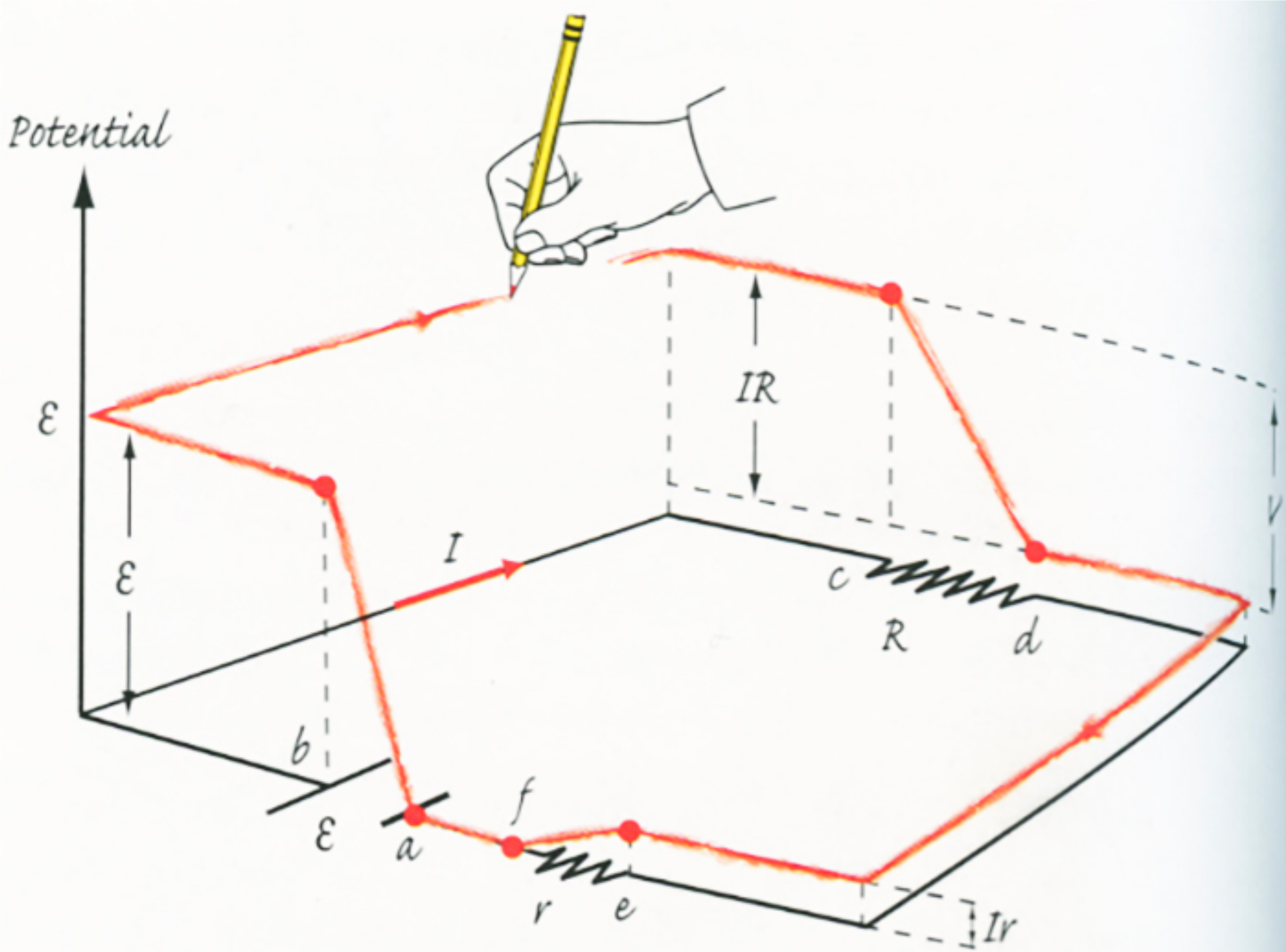

Current
0.30 A
12.0 V
40.0 Ω

Unit 23 – Session 3 (cont.)

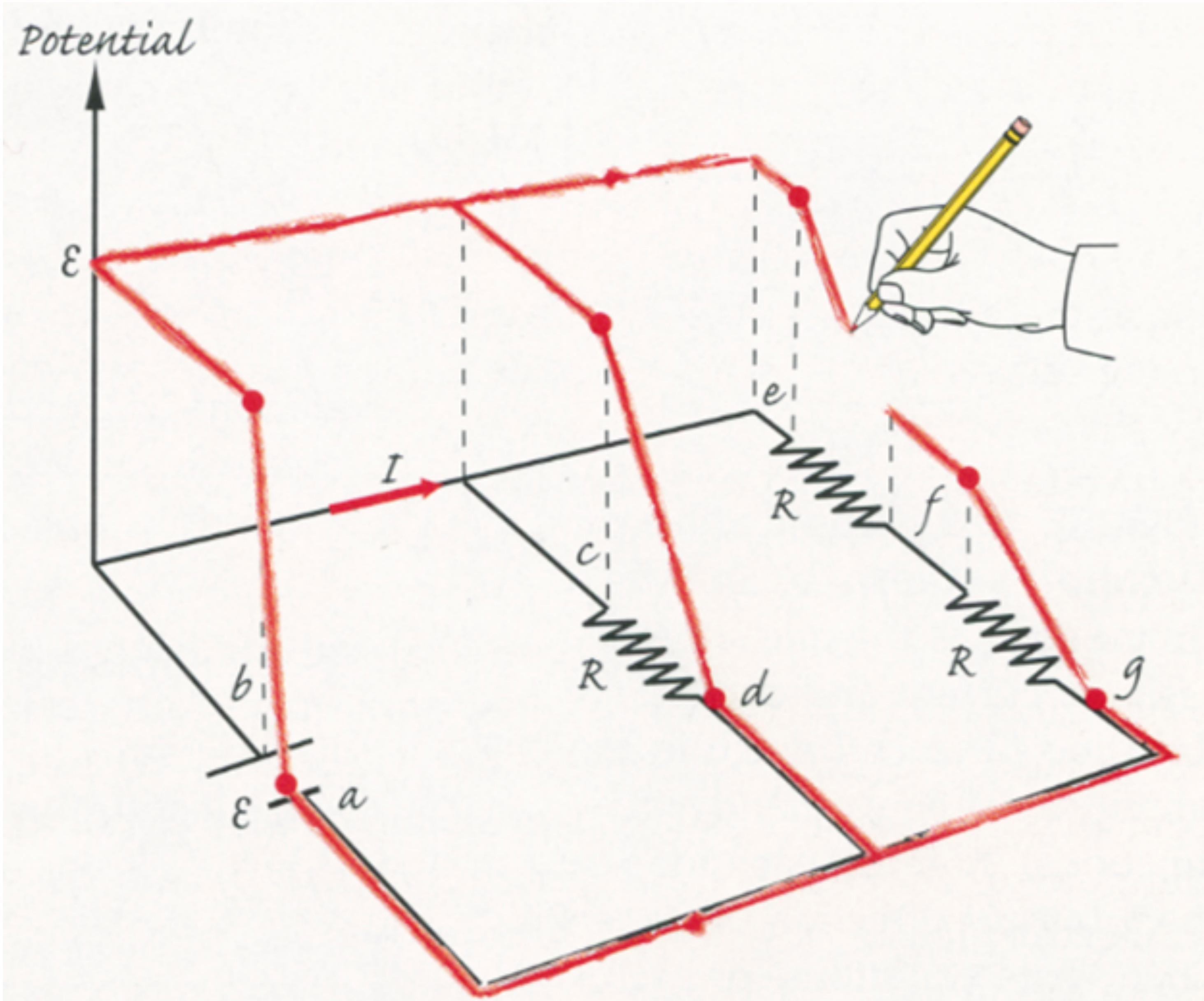
Today, we are going to develop a set of laws – Kirchhoff's Laws – that will allow us to analyze these more complex circuits. The laws are very powerful, and can even be used as an alternative method to analyze parallel/series circuits that we could easily simplify.

The laws are based on two conservation laws:

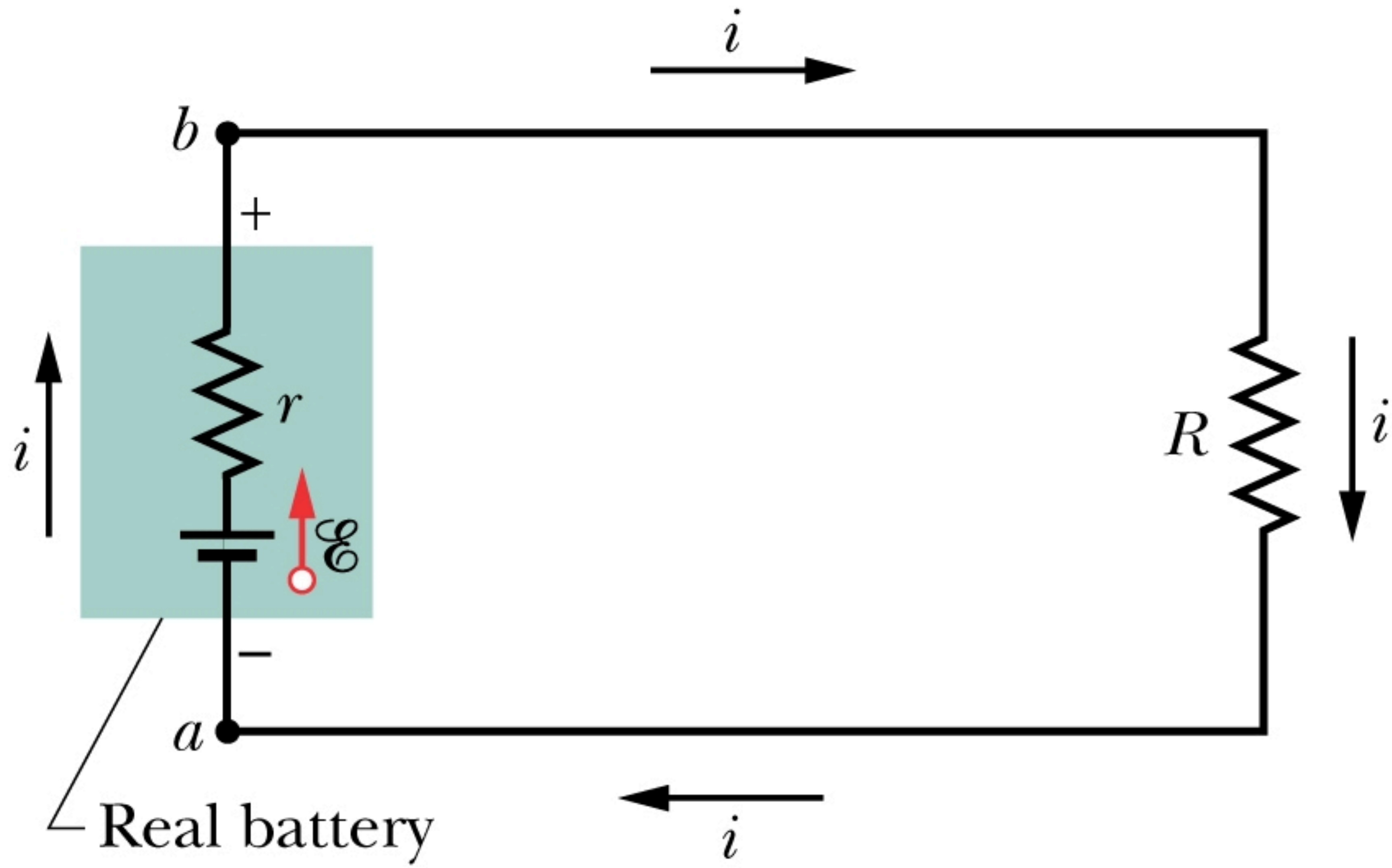
- Conservation of charge.
- Conservation of energy.



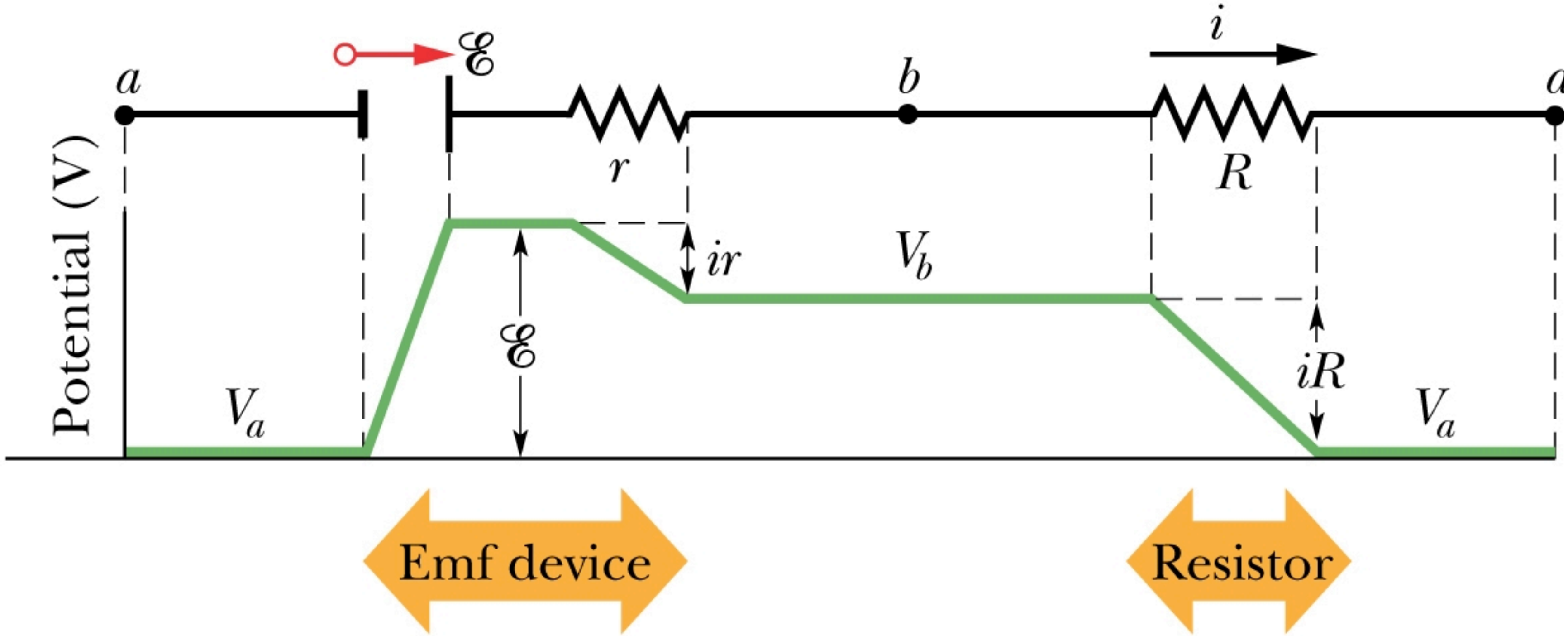
Potential
ℰ
ℰ
I
IR
c
R
d
V
b
ℰ
a
f
r
e
Ir

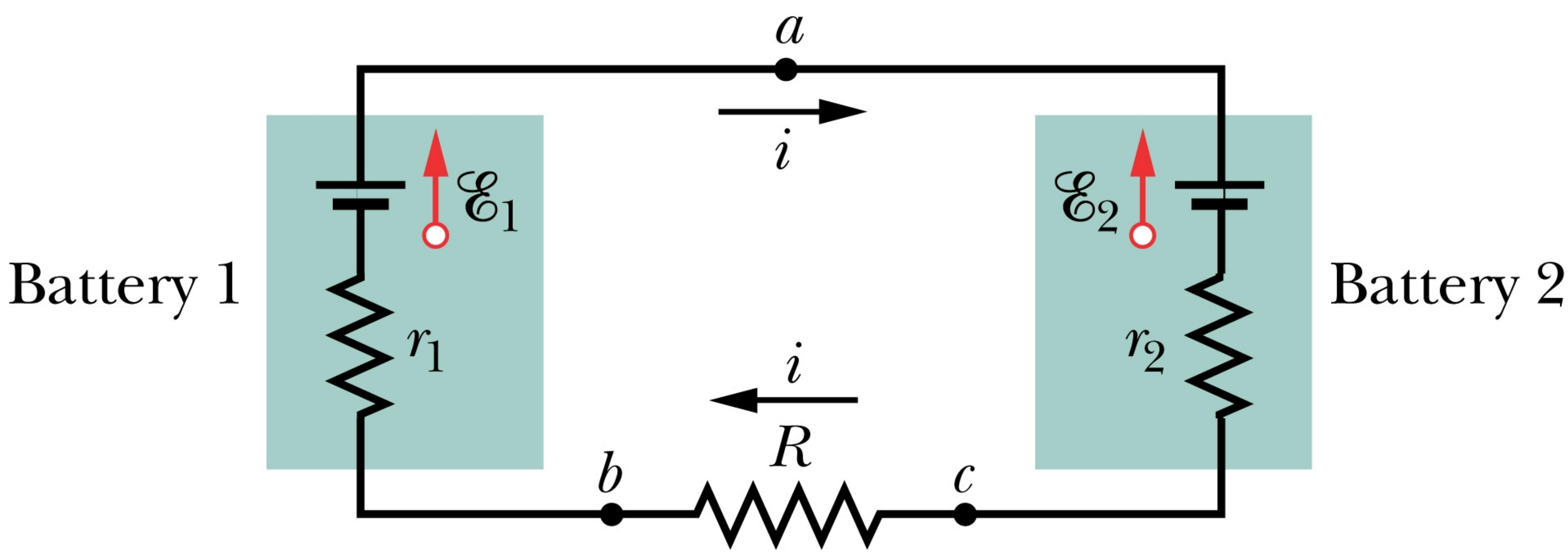


Potential
ℰ
I
e
R
f
c
R
b
d
R
g
ℰ
a

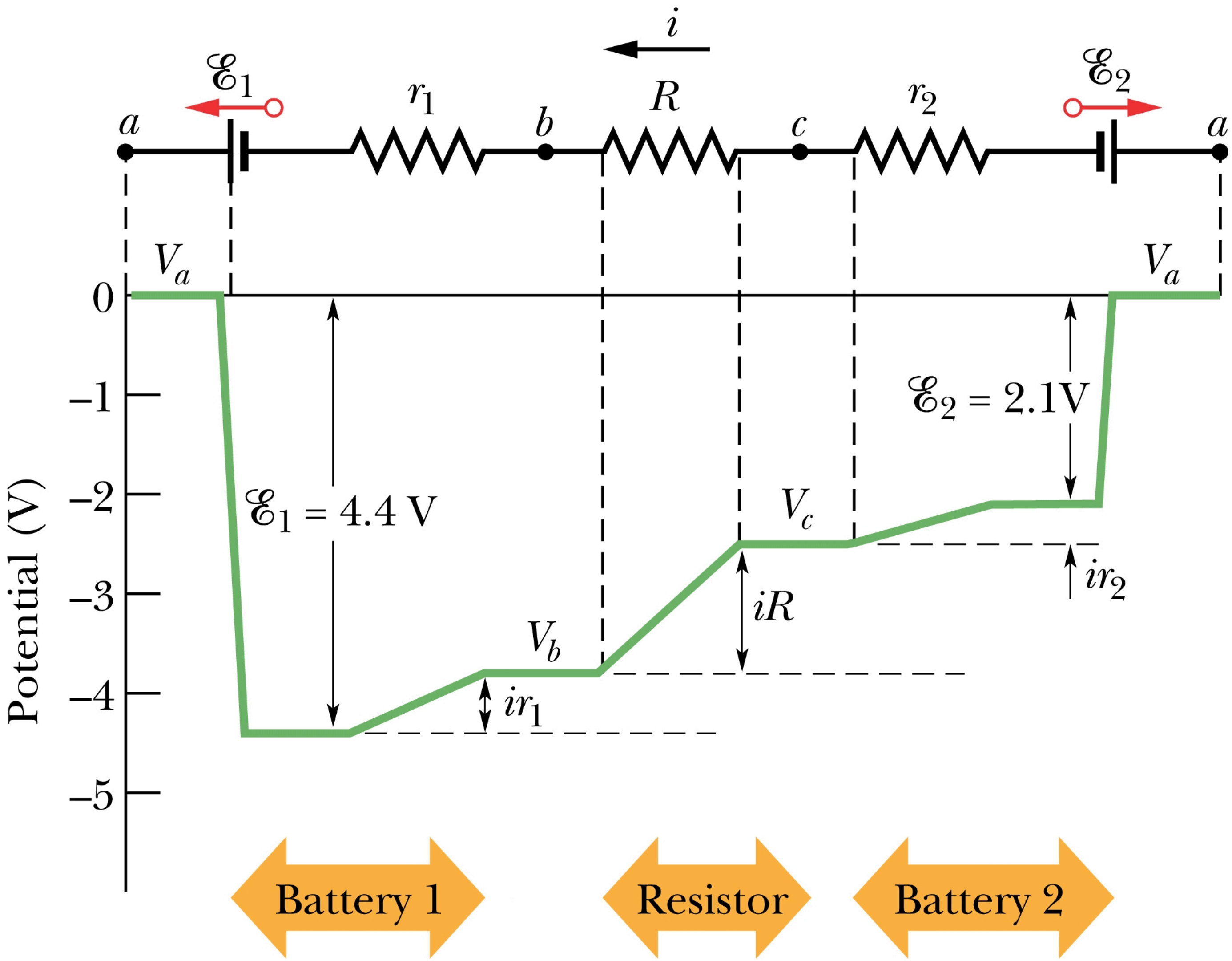


i
b
+
r
i
ℰ
R
i
−
a
i
Real battery


a
ℰ
r
b
i
R
a
Potential (V)
ir
V_b
ℰ
iR
V_a
V_a
Emf device
Resistor



a
i
$\mathscr{E}_1$
$\mathscr{E}_2$
Battery 1
Battery 2
r_1
r_2
i
R
b
c



i
ℰ1
r1
R
r2
ℰ2
a
b
c
a
Va
Va
0
−1
−2
−3
−4
−5
Potential (V)
ℰ1 = 4.4 V
ℰ2 = 2.1V
Vb
Vc
ir1
iR
ir2
Battery 1
Resistor
Battery 2

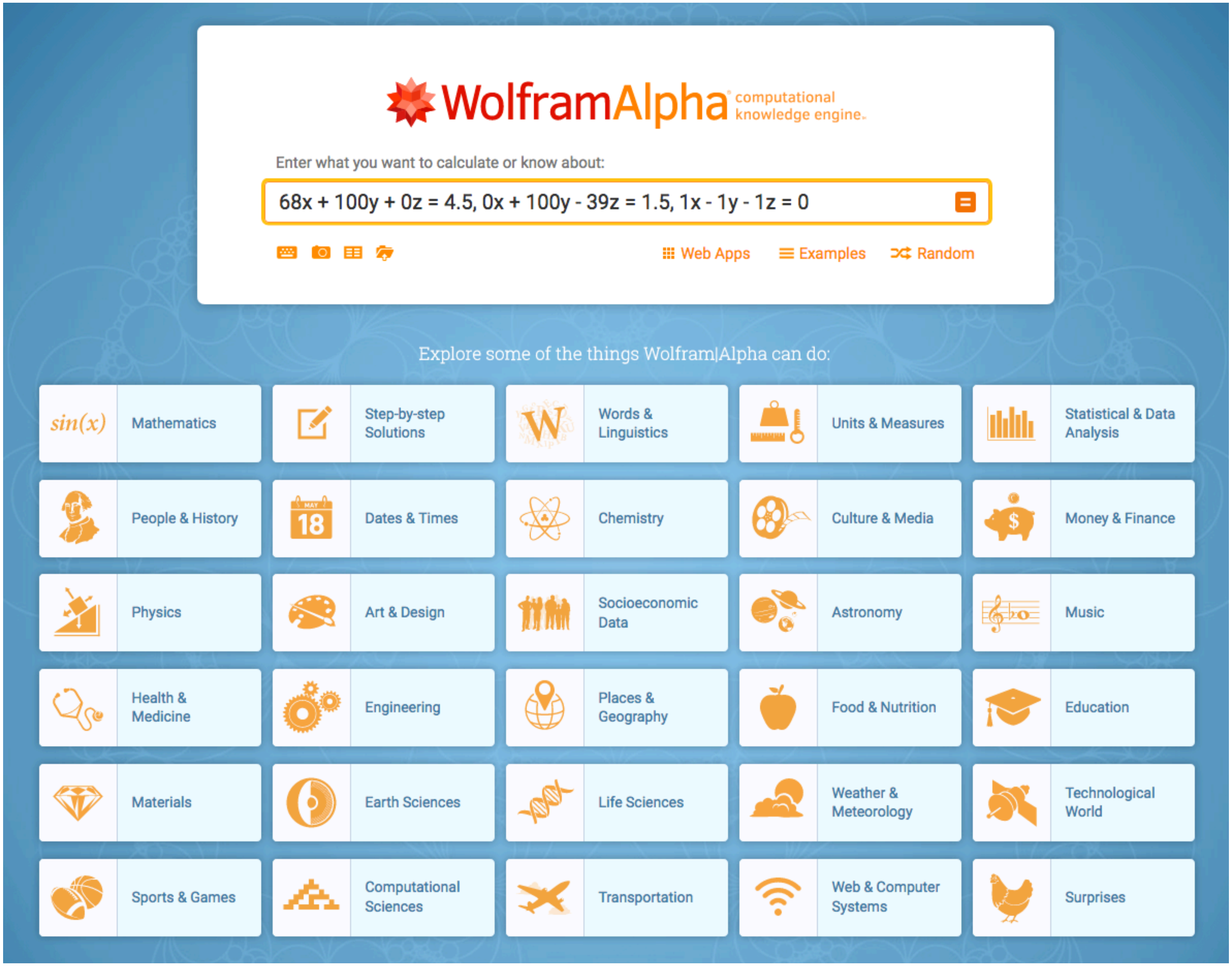
Activity 23.11.1.

For many circuits, we don't know beforehand what the currents are in each branch, or even what direction they might be going in. In order to write Kirchhoff's equations, we have to pick a current direction, even if it isn't the direction the current is actually flowing in.

Just pick a current direction for each branch, and move on.

Just pick a loop direction for each loop, and move on.

- Part **a**. – In the example, they arbitrarily picked the current in the middle branch, i_2, to be flowing from top to bottom, and arbitrarily picked the 2 loop directions to be clockwise. Here, they want you to have the current in the middle branch flowing from bottom to top, and call it by a different name, i_2' (note the prime, it looks kind of funky in the figure.). They also want you to have the 2 loop directions be counterclockwise. (Hint: These changes will just change most (but not all) of the signs in equations 23.1, 23.2, and 23.3)
- Part **b**. – Take the 3 equations you came up with in Part **a.** and replace i_2' with $-i_2$ in each one. Now rearrange them until they look like equations 23.1, 23.2, and 23.3.


WolframAlpha computational knowledge engine.
Enter what you want to calculate or know about:
68x + 100y + 0z = 4.5, 0x + 100y - 39z = 1.5, 1x - 1y - 1z = 0
Web Apps
Examples
Random
Explore some of the things Wolfram|Alpha can do:
sin(x)
Mathematics
Step-by-step Solutions
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Units & Measures
Statistical & Data Analysis
People & History
Dates & Times
Chemistry
Culture & Media
Money & Finance
Physics
Art & Design
Socioeconomic Data
Astronomy
Music
Health & Medicine
Engineering
Places & Geography
Food & Nutrition
Education
Materials
Earth Sciences
Life Sciences
Weather & Meteorology
Technological World
Sports & Games
Computational Sciences
Transportation
Web & Computer Systems
Surprises

WolframAlpha computational knowledge engine.

68x + 100y + 0z = 4.5, 0x + 100y - 39z = 1.5, 1x - 1y - 1z = 0

Web Apps | Examples | Random

Input:

$\{68x + 100y + 0z = 4.5,\ 0x + 100y - 39z = 1.5,\ 1x - 1y - 1z = 0\}$

Open code

Result:

$\{68x + 100y = 4.5,\ 100y - 39z = 1.5,\ x - y - z = 0\}$

Solution:

Step-by-step solution

Try it!

$x \approx 0.0356126, \quad y \approx 0.0207834, \quad z \approx 0.0148292$

Alternate forms:

$\{x + 1.47059y = 0.0661765,\ y = 0.39z + 0.015,\ x = y + z\}$

$\left\{y = 0.045 - \frac{17x}{25},\ z = \frac{100y}{39} - 0.0384615,\ z = x - y\right\}$

$\{4(17x + 25y) = 4.5,\ 100y - 39z = 1.5,\ x - y - z = 0\}$

Alternate form assuming x, y, and z are positive:

$\{x + 1.47059y = 0.0661765,\ y = 0.39z + 0.015,\ x = y + z\}$

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