Unit 20 – Session 2

Last time, we looked at two separate concepts:

- Field lines
  - # of lines is directly proportional to the magnitude of the charge.
- Flux, $\Phi^{elec} = \int \vec{E} \cdot d\vec{A}$
  - Flux is directly proportional to the # of lines at the surface of an area, pointing outward (positive flux) or pointing inward (negative flux).

When we introduced the concept of work, $W \equiv \int \vec{F} \cdot d\vec{s}$, last semester, it wasn't that useful to us until we related it to another concept: change in kinetic energy, $\Delta K \equiv \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$.

$$W = \Delta K \quad \text{or} \quad \int \vec{F} \cdot d\vec{s} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Activity 20.5

Here, we are going to combine the concepts of field lines and flux together to get a useful relationship, like we did for work and kinetic energy.

Our previous exploration of flux was for an open surface (a sheet of paper) – however, the concept of flux won't be useful to us unless we look at a 3-dimensional closed surface, with a volume inside our surface (like a sphere, or a box, or a cylinder).

Some terms and "rules":

1) You can pick any surface area shape you want, but it must be a closed surface area (encloses a volume).

2) The surface area is called a Gaussian surface

3) The unit area vectors of the surface always point outward.

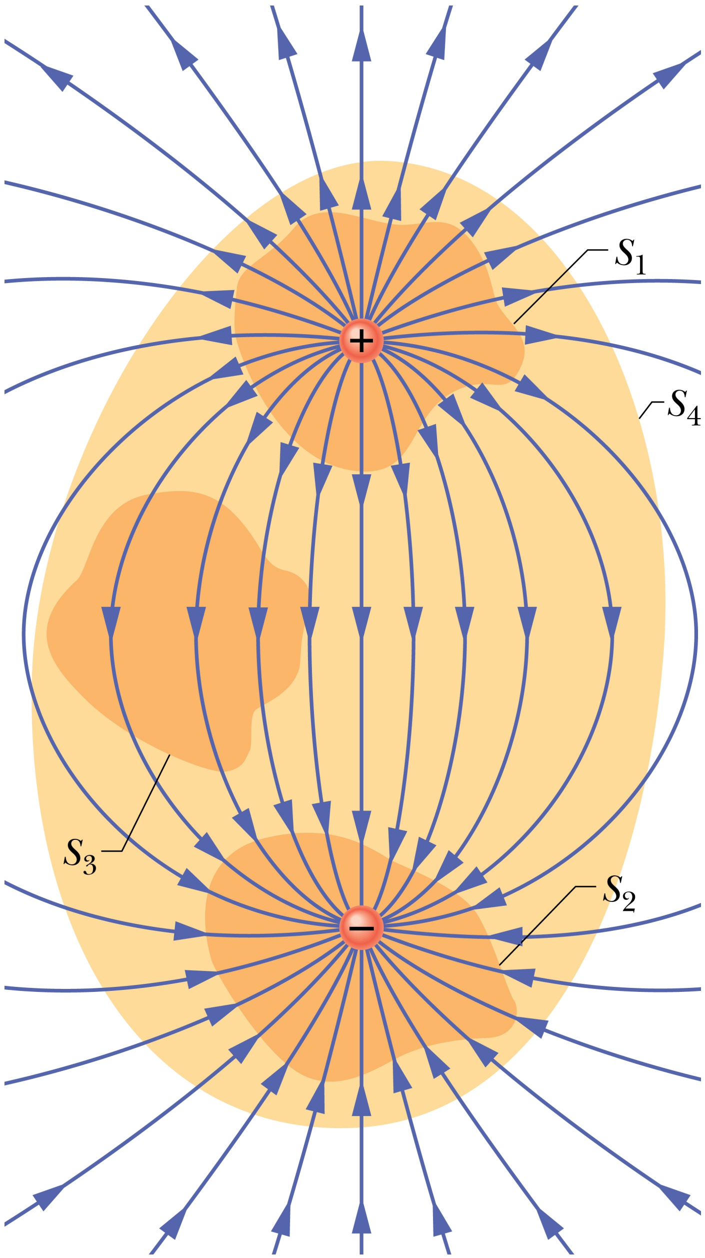
4) $\vec{E}$ field lines pointing from inside to outside $\Rightarrow$ $+\Phi^{elec}$
   $\vec{E}$ field lines pointing from outside to inside $\Rightarrow$ $-\Phi^{elec}$

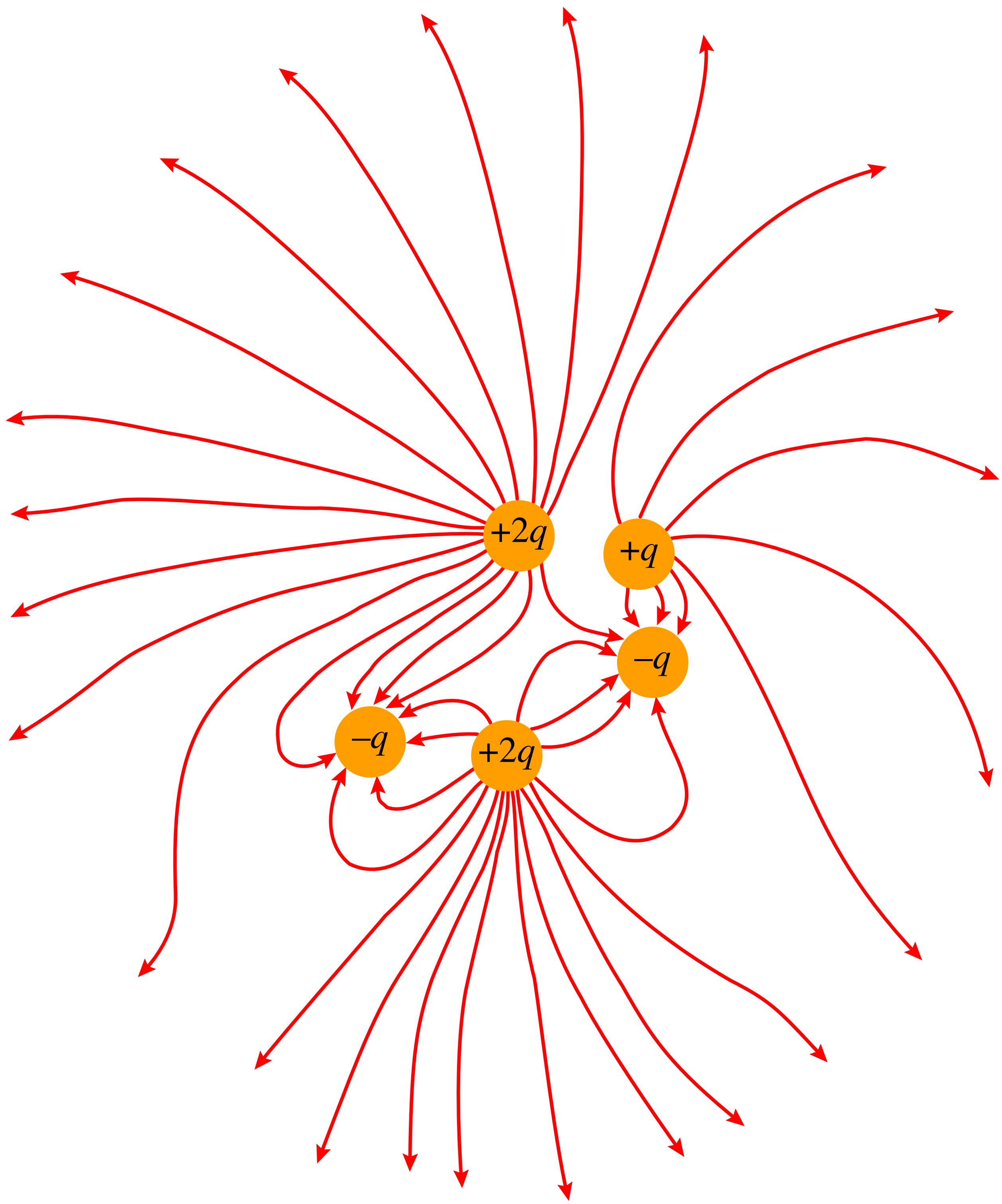
Since 3-dimensional surfaces are hard to draw on paper, we will explore the relationship between electric field lines and flux using an abstract world called Flatland:

Instead of a closed surface area surrounding a volume

$\Rightarrow$ a closed boundary line surrounding an area.

An example (before we do Activity 20.5.1).

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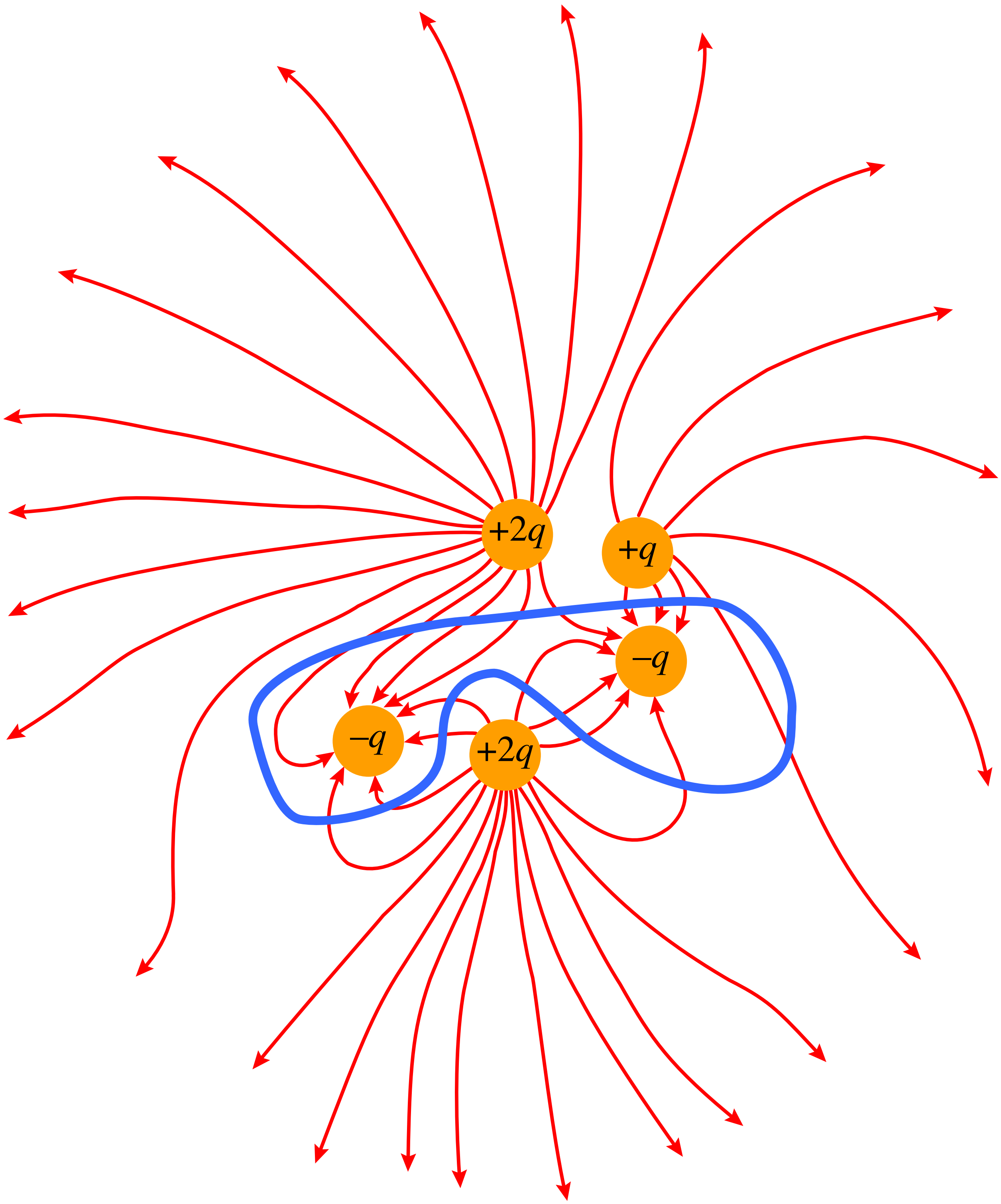
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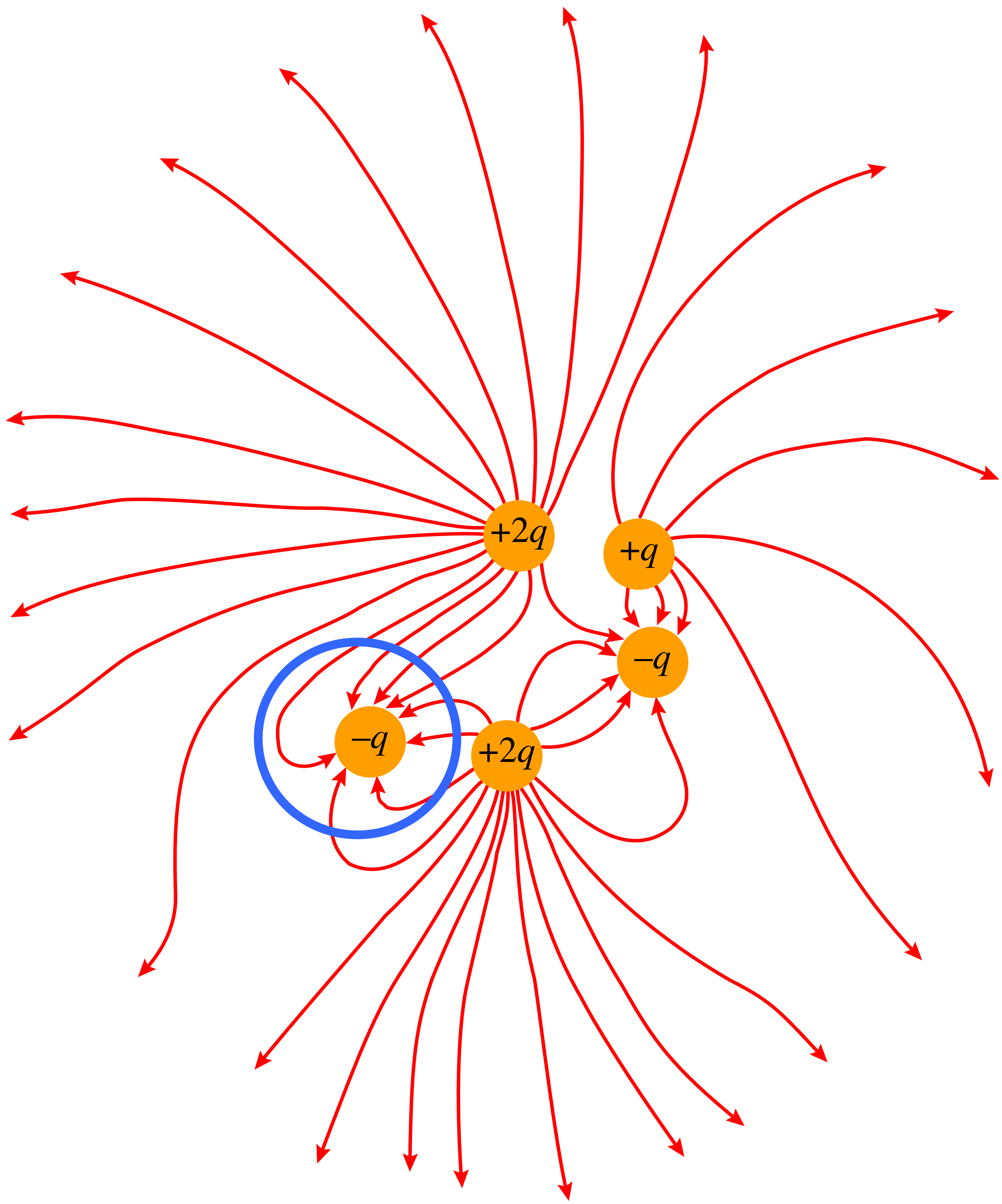

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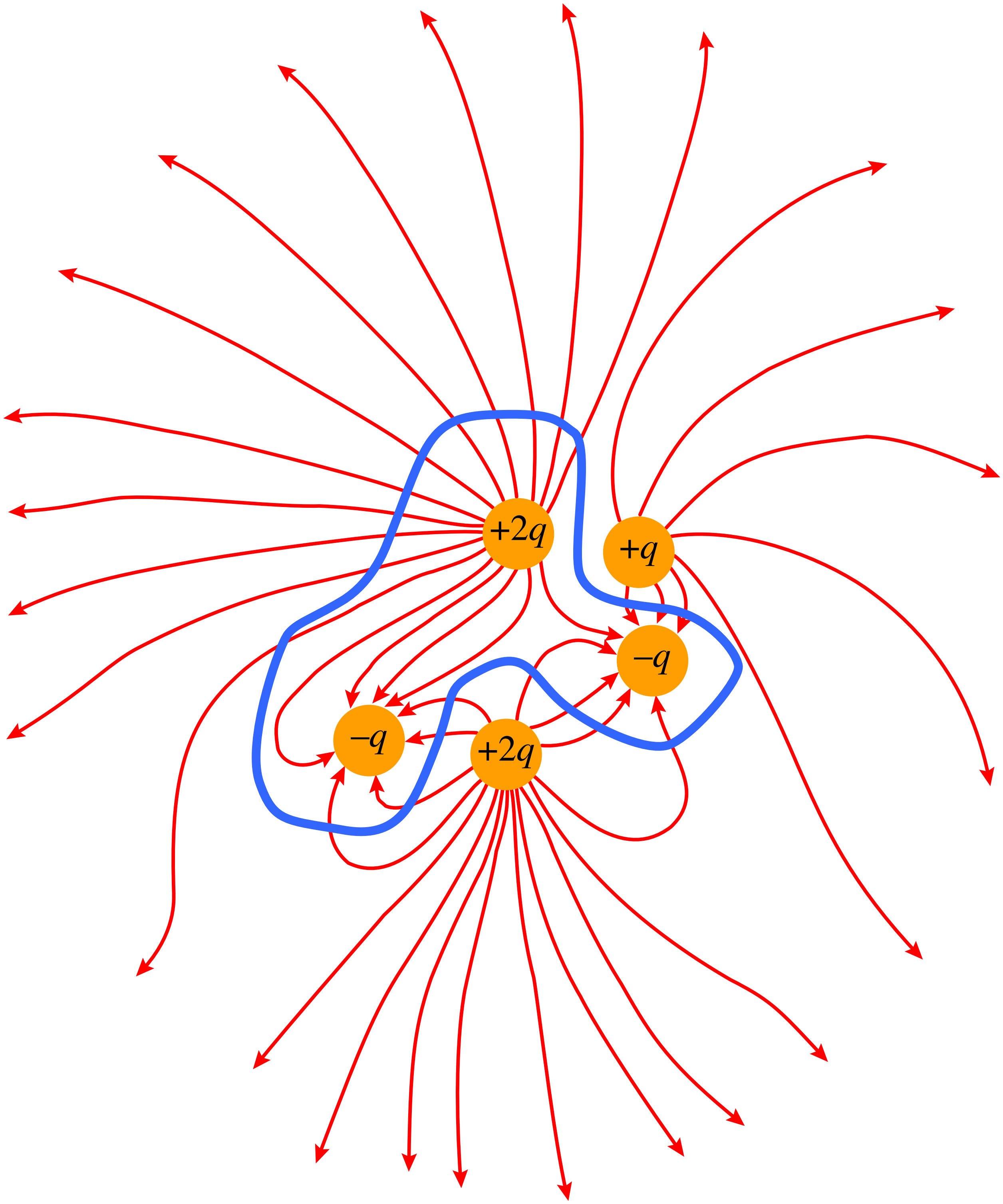
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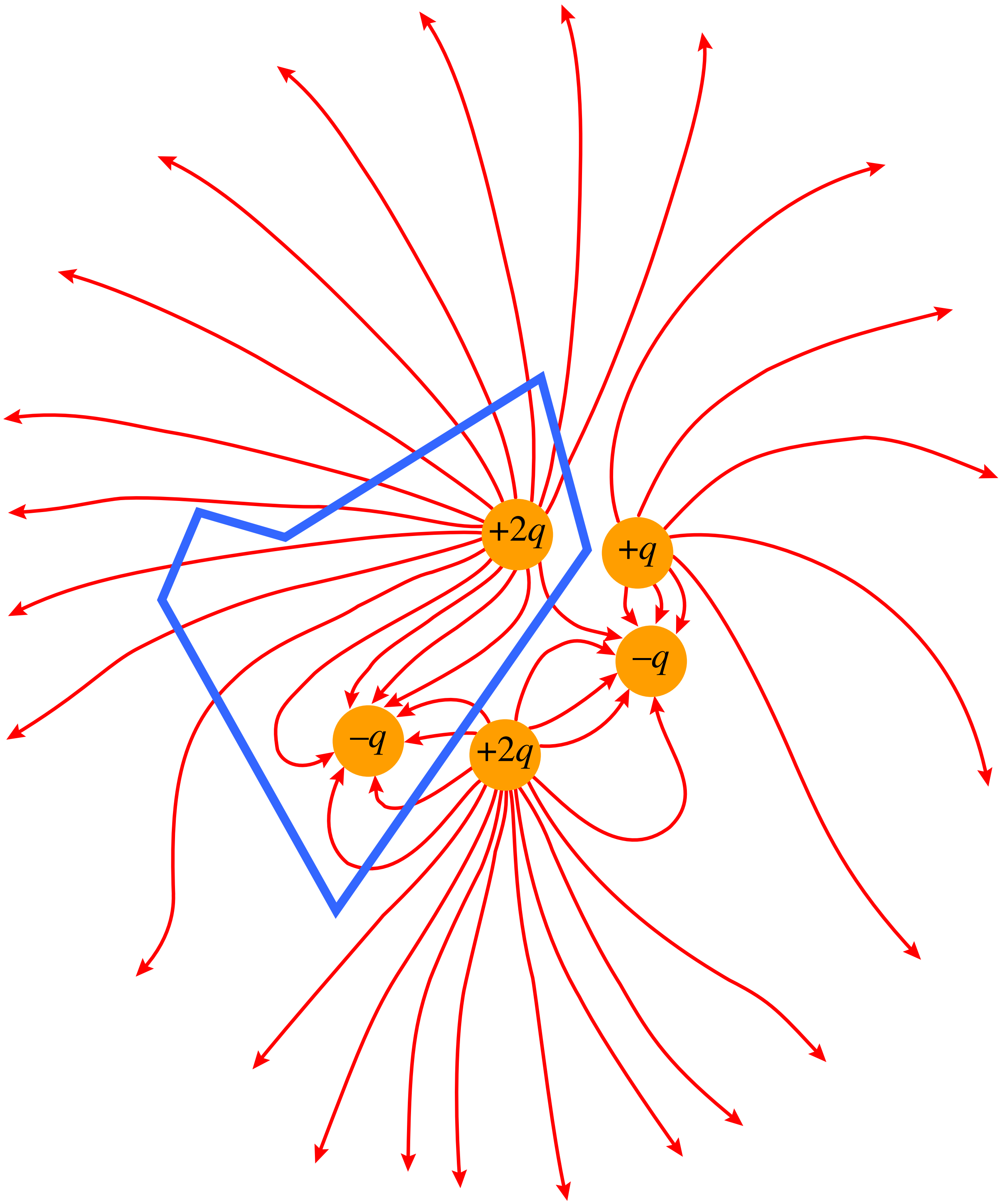

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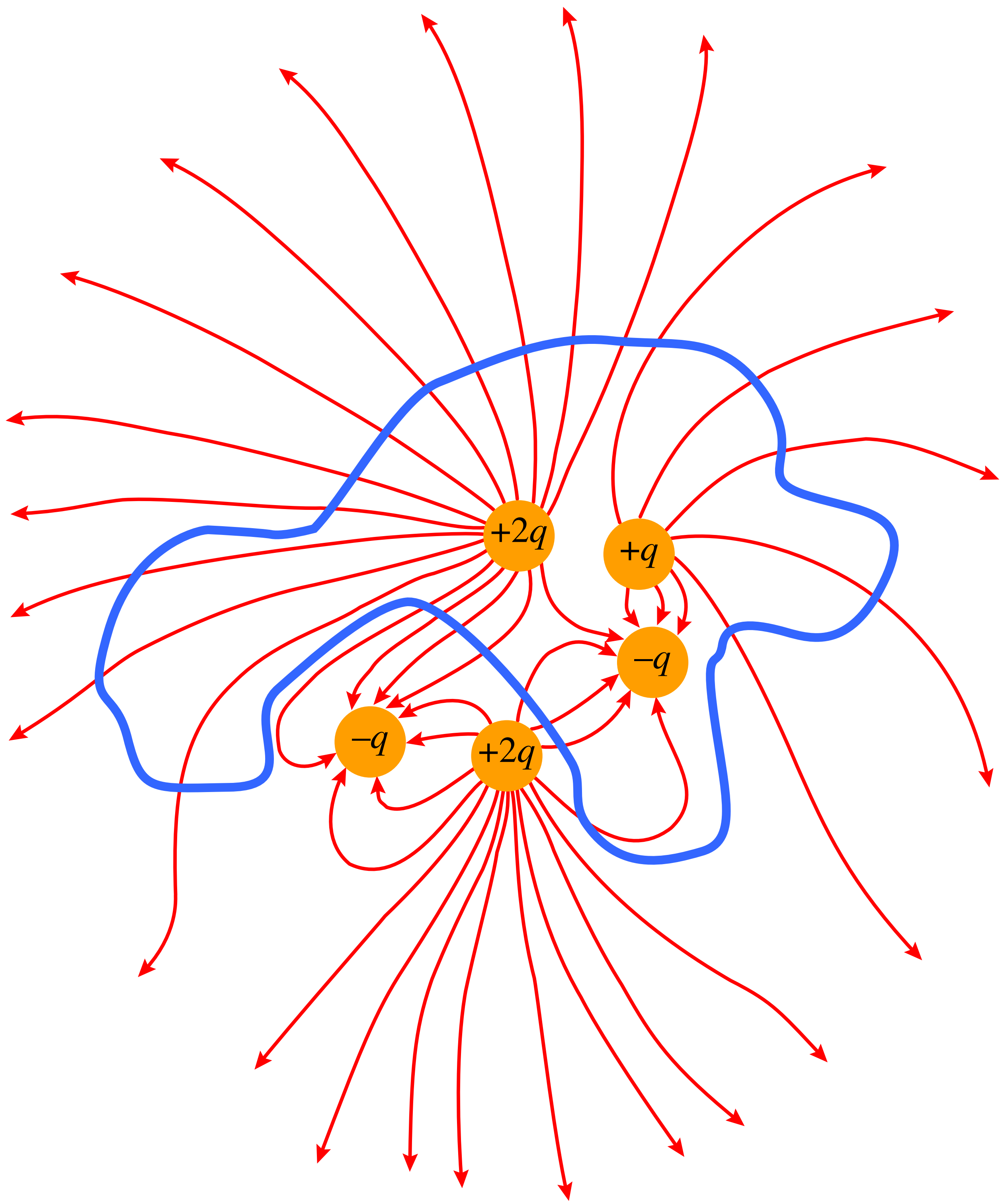
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Activity 20.5.2.

Hopefully, you concluded a few things from your observations:

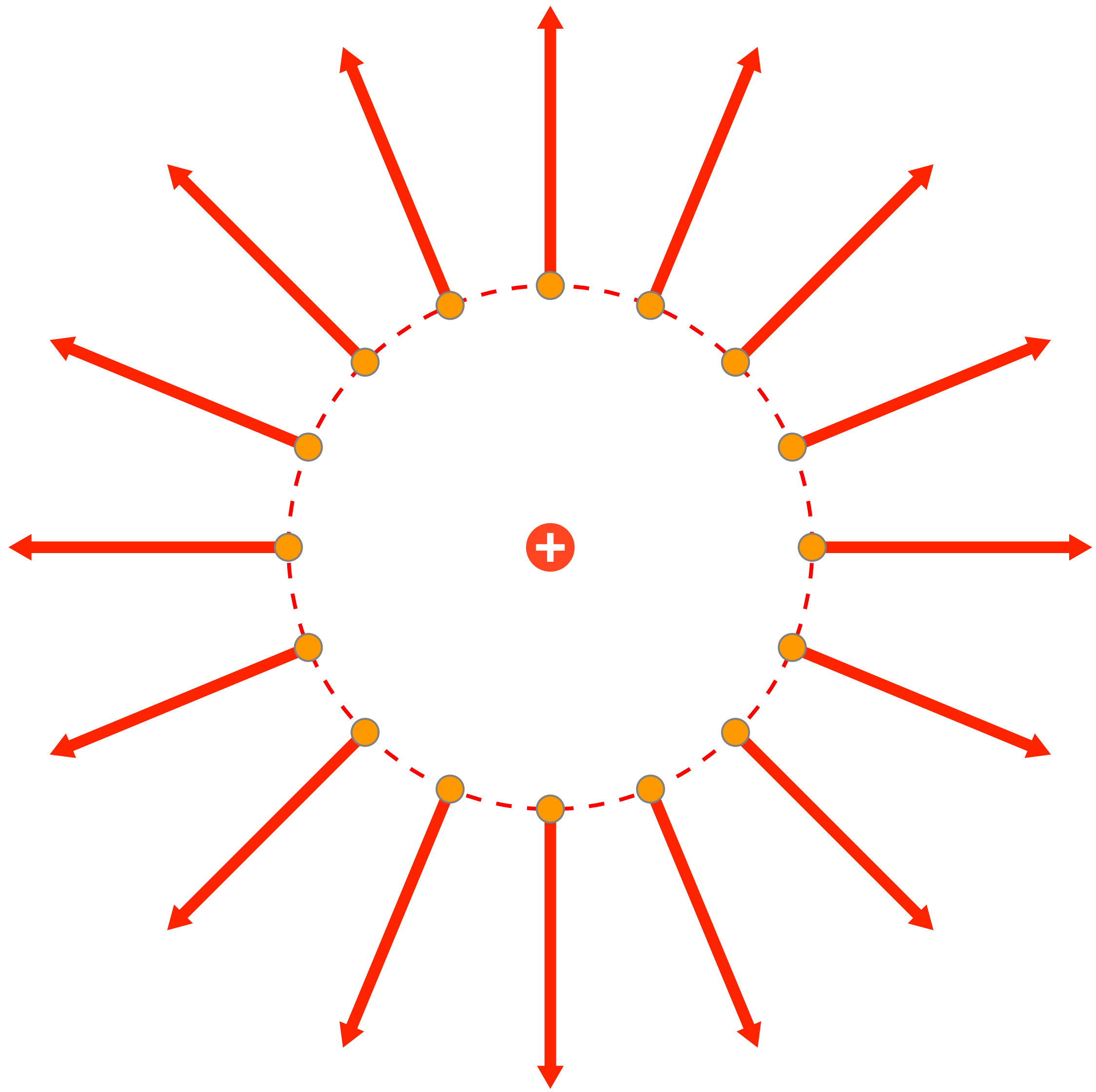
- The flux at the boundary of a closed surface is directly proportional to the total amount of charge enclosed by that surface:
    - $\Rightarrow \quad \Phi^{elec} = (constant)\, q^{enclosed}$
        - When we do the 3rd session for this Unit (20.8 & 20.9) – we will find there that the *constant* is equal to $4\pi k$, where *k* is Coulomb's constant. It is sometimes written instead as $constant = 4\pi k = \frac{1}{\varepsilon_0}$ (ε_0 is pronounced "epsilon zero" or "epsilon naught").
- This relationship between the net flux at the boundary of a closed surface and the enclosed charge is called Gauss' Law.
    - $\Longrightarrow$ $$\oint \vec{E} \cdot d\vec{A} = 4\pi k q^{enc}$$ $$\oint \vec{E} \cdot d\vec{A} = \frac{q^{enc}}{\varepsilon_0}$$

Activity 20.5.2 (continued)

There are a couple of things to note about Gauss' Law

- If you know or are given the total amount of charge inside the closed surface, the right side of the equation ($4\pi k q^{enc}$) is easy to calculate.
- The left side of the equation is not easy to calculate unless we limit ourselves to a few special cases:
  - The magnitude of the electric field ($|\vec{E}|$) is the same everywhere on the closed surface.
  - The angle (θ) between the electric field vector ($\vec{E}$) and the area vector ($d\vec{A}$) is always the same everywhere on the closed surface.

$$\oint \vec{E} \cdot d\vec{A} = EA\cos 0^\circ \text{ is easy to calculate.}$$



$\oint \vec{E} \cdot d\vec{A}$ is not easy to calculate.