d. Use a ruler or video analysis software to measure the average time, Δt_x, it takes the atom to move from the left wall of the box to the right wall. Also determine Δt_y.

e. Calculate the magnitude of the x and y components of velocity.

f. Use a stopwatch and ruler or video analysis software to measure the total length of the straight path of the molecule in centimeters as it moves between a collision with the right wall and a collision with the bottom wall. Also determine the time between the collisions. Use your data to compute the speed of the molecule.

g. How does the measured speed of your molecule compare to that determined by the equation $v^{\text{total}} = \sqrt{\left(v_x^{\,2} + v_y^{\,2}\right)}$?

h. Suppose the box were a three-dimensional container. Write the expression for v^{total} in terms of the x, y, and z components of velocity. **Hint:** This is an application of the 3-dimensional Pythagorean theorem.

i. We would like to find the average kinetic energy of each molecule. Since the kinetic energy of a molecule is proportional to the square of its total speed, you need to show that if *on the average* $v_x^{\,2} = v_y^{\,2} = v_z^{\,2}$, then $(v^{\text{total}})^2 = 3v_x^{\,2}$.

j. Assume that the interaction time between the molecule and the wall is negligible. If the molecule bounces back at the same speed in the x-direction, show that the total time, Δt, it takes to return to the left wall is $\Delta t = 2\,\Delta t_x$.

k. If the collisions with the wall perpendicular to the x-direction are elastic, show that the x-component force exerted on that wall for each collision is just $F_x = 2m(v_x)/(2\Delta t_x)$ where m is the mass of the particles and $2\Delta t_x$ the mean interval between collisions with the wall. (**Hint:** Think of the vector form of Newton's second law in which force is defined in terms of the change in momentum per unit time so that $F_x = \Delta p_x / \Delta t$.) **Warning:** Physicists often use the same symbol to stand for more than one quantity. In this case, note that Δp_x (where "p" is in lowercase) indicates the change in momentum, and not pressure change.

l. Substitute the expression from part a. for Δt_x to show that

$$F_x = \frac{mv_x^2}{L_x}$$

m. For simplicity, let us assume that we have a cubical box so that the length, width, and height are all the same so that $L = L_x = L_y = L_z$. Hence, the volume of the box is the cube of its length. In other words, $V = L^3$. Show that the pressure on the wall perpendicular to the x-axis caused by the force F_x due to one atom is

$$P = \frac{mv_x^2}{V}$$

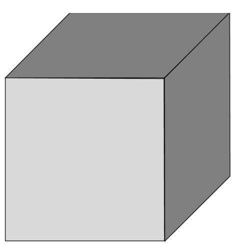

Fig. 17.9. The volume of the box is $V = L_x L_y L_z = L^3$.

Note: For simplicity we use a cubical box. You should be able to convince yourself that you will obtain the same result for a box of dimensions L_x, L_y, and L_z.

n. Let's say that there are not one but N molecules in the box. What is the pressure on the wall now?

b. In general, a gas that is made up of molecules can store internal energy by rotating or vibrating. For an ideal gas of point particles like the atoms we have considered, the only possible form of internal energy is the sum of the translational kinetic energies of the atoms. If we can ignore potential energy due to gravity or electrical forces, then the internal energy, E^{int}, of a gas of N particles is $E^{\text{int}} = N\langle K\rangle$. Use this to show that for an ideal gas of point particles, E^{int} *depends only on N and T*. Derive the equation that relates E^{int}, N, and T for an ideal gas. Show the steps.

The microscopic and the macroscopic definitions of temperature are equivalent. The microscopic definition of temperature that you just derived is fundamental to the understanding of all of thermodynamics!

A Note About Internal Energy

The microscopic model for an ideal gas is that of a collection of point particles that undergo perfectly elastic collisions. We noted that the internal energy in an ideal gas is just the sum of the translational kinetic energies of its particles.

The molecules in a real gas often have structures that can rotate and vibrate. Thus, several types of energy must be taken into account in determining internal energy.

In *liquids and solids* atoms and molecules exert forces on each other and are part of a complex system that has both potential and kinetic energy.