b. In an *isothermal* compression, the gas is kept at a constant temperature by thermal energy transfer to its surroundings. Therefore ΔT and ΔE^{int} are both zero in an isothermal compression. Find an expression relating Q and W and another expression relating P and V during an isothermal compression. **Hint:** Use both the first law of thermodynamics and the ideal gas law.

c. The Boyle's law experiment that some of you carried out in an earlier unit was in fact an isothermal compression, since the syringe was always in thermal equilibrium with the room. Are the Boyle's law measurements you or your classmates made consistent with what you derived above? Explain.

Another type of process that can occur during the expansion or compression of a gas is an *adiabatic* change. An adiabatic process is defined as one in which a system does not exchange thermal energy with its surroundings so that $Q = 0$ during the process. This can be brought about either by carefully insulating the system so that no thermal energy exchange is possible, or by carrying out the process so rapidly that thermal energy transfer does not have time to take place. What happens to an ideal gas if it is compressed adiabatically? We would like you to show on a step-by-step basis, outlined in Activity 18.3.2 through Activity 18.4.2 below, that for an ideal gas undergoing an adiabatic expansion the following expression can be used to describe the relationships between an initial volume and temperature and a final volume and temperature.

$$\frac{3}{2}\frac{dT}{T} + \frac{dV}{V} = 0 \quad \text{so that} \quad T_2^{3/2}V_2 = T_1^{3/2}V_1$$

Note: The exponent of 3/2 only holds for an ideal monatomic gas. For a "real" gas the exponent will be different.

18.3.2. Activity: Adiabatic Compression of a Gas

a. The first step: In Activity 18.3.1 you showed that the change in the internal energy of an ideal gas is given by $\Delta E^{\text{int}} = (3/2)Nk_B\Delta T$. Use the first law of thermodynamics to find a relationship between the work done when an ideal gas is compressed adiabatically by an amount ΔV (with no change in pressure) and the change in the temperature of the gas, ΔT. In particular, show that $(3/2)Nk_B\Delta T = -P\Delta V$. **Hint:** For a small change in volume at constant pressure the amount of work done, W, is given by $W \approx P\Delta V$.

b. Next you can use the ideal gas law and the relationship you just derived to show that for small temperature changes the fractional change in the temperature of an ideal gas, $\Delta T/T$, can be related to the fractional change in volume, $\Delta V/V$, by

$$\frac{3}{2}\frac{\Delta T}{T} = -\frac{\Delta V}{V} \quad \text{or} \quad \frac{3}{2}\frac{\Delta T}{T} + \frac{\Delta V}{V} = 0$$

18.4. MATHEMATICAL INTERLUDE: LN(X) AND $\int$DX / X

In the study of adiabatic processes we sometimes encounter equations with some thermodynamic variable raised to a power, for example, P^{γ}. It is often useful in manipulating such equations to take logarithms. Some of you may already be familiar with common or base 10 logarithms. Usually denoted $\log(x)$ or $\log_{10}(x)$, the logarithm of x is that power to which the base number of 10 must be raised in order to obtain x. For example, 10 must be raised to the third power to get 1000, since $10^3 = 1000$; consequently $\log_{10}(1000) = 3$. Put in terms of symbols, if $x = 10^r$ then $\log_{10}(x) = r$. Logarithms of numbers follow certain rules that make them helpful calculation aids. For example, the relations:

$$\log_{10}(A \times B) = \log_{10}(A) + \log_{10}(B)$$
$$\log_{10}(A / B) = \log_{10}(A) - \log_{10}(B)$$
$$\log_{10}(A^n) = n\log_{10}(A)$$

hold true for any positive numbers A and B and exponent n.

It is possible to use a base other than 10 to set up a system of logarithms. Frequently in mathematics, base e logarithms ($\ln(x)$) are used, where e = 2.7182818294 … and is a transcendental number. For base e or *natural*

Now we are back to proving that

$$T_2^{3/2} V_2 = T_1^{3/2} V_1 \tag{18.2}$$

18.4.2. Activity: *T* vs. *V* for Adiabatic Expansions

Combine the results you just obtained in Activities 18.3.2b and 18.4.1c to show that, if a gas of initial temperature T_1 and volume V_1 is compressed adiabatically to a final volume V_2 with final temperature T_2, then

$$T_2^{3/2} V_2 = T_1^{3/2} V_1$$

You can do this by integrating both sides of the equation given by

$$\frac{3}{2}\frac{dT}{T} + \frac{dV}{V} = 0$$

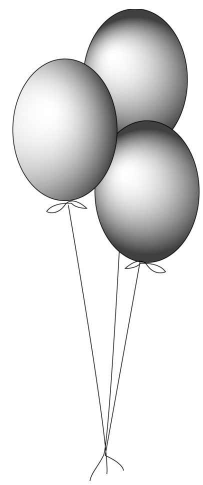

Fig. 18.1.

Note: Strictly speaking, this result only holds for ideal monatomic gases composed of single molecules such as helium. A similar equation holds for other gases at modest densities.

18.5. THE FIRE SYRINGE AND THE RAPID COMPRESSION OF AIR

A device known as a *fire syringe* allows a rapid compression of air in a small glass tube that is inside a safety tube of Plexiglas. If pushed very hard, the piston in the glass tube can be forced almost down to the end of the straight-walled section of the tube. If this is done rapidly, the compression can be nearly adiabatic. Air is not a monatomic gas, but the formulas derived above work well enough. As you can tell from the equation you derived in the last activity, the air in the fire syringe should increase in temperature as its volume decreases. Examine a fire syringe and make some reasonable assumptions about the initial and final volumes of air in the chamber. You can then calculate the approximate final temperature of the compressed air. Finally, you can attempt to ignite a tiny piece of tissue paper with the fire syringe. For this activity you will need:

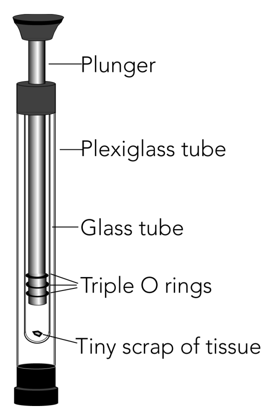

Fig. 18.2. A fire syringe that allows a rapid compression of trapped air to ignite a piece of paper.

- 1 fire syringe
- 1 ruler
- 1 glass thermometer
- 1 piece of tissue paper, approx. 0.5 mm on a side

Recommended Group Size:	2	Interactive Demo OK?:	Y

18.5.1. Activity: The Fire Syringe—Fahrenheit 451

a. Before approximating the final temperature of air in the syringe, estimate the following quantities:

Initial length of air column L_1 = ________ cm

Final length of air column L_2 = ________ cm

Inner radius of the tube R = ________ cm

Initial volume V_1 = ________ cm^3

Final volume V_2 = ________ cm^3

Initial temperature T_1 = ________ K

b. Calculate the final temperature in the cylinder in Kelvin.

Final Temperature T_2 = ________ K

c. Compare this to the "flash point" or burning temperature of paper, which is 451°F* What do you expect to happen to the tissue paper in the fire syringe when the plunger is pushed down rapidly?

d. Put on safety gloves and carry out the fire syringe experiment by rapidly and forcefully depressing the plunger. What happens?

e. Why doesn't the tissue paper catch fire when you compress the air slowly?

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* The paper flashpoint of 451 degrees Fahrenheit is well known to readers of Ray Bradbury's famous science fiction novel, *Fahrenheit 451,* about book burning.