gas is the same as the energy transferred to it. This can be seen from the first law of thermodynamics since

$$\Delta E^{\text{int}} = Q - W = Q - P\Delta V = Q \left(\text{Since } \Delta V = 0\right)$$

For an ideal gas we can write the internal energy as

$$E^{\text{int}} = \frac{3}{2}NkT = \frac{3}{2}nRT \tag{18.8}$$

where N is the number of molecules, $n = N/N_A$ is the number of moles of the gas, and N_A is Avogadro's number. Because the volume of our gas remains constant,

$$\Delta E^{\text{int}} = Q = C_V n \Delta T$$

or

$$C_V = \frac{1}{n}\left(\frac{\Delta E^{\text{int}}}{\Delta T}\right) = \frac{3}{2}R \tag{18.9}$$

where C_V is the *molar heat capacity at constant volume* and R is the universal gas constant. We can define C_V for a non-ideal diatomic gas using the same approach, but it will not have the value 3/2 R.

In order to understand the Carnot cycle as an ideal heat engine cycle, we must explore the nature of adiabatic expansions and the work associated with them. Adiabatic expansions are a function of the ratio of the molar heat capacity at constant volume and that at constant pressure. It can be shown mathematically that the relationship between these two heat capacities is

$$C_P = C_V + R \tag{18.10}$$

If this equation is valid, then obviously C_P is greater than C_V. Another way of saying this is that when a given amount of thermal energy is transferred to a gas, the temperature of the gas will rise more when the volume is held constant than when the pressure is held constant. How come? This relationship can be explained using kinetic theory.

For simplicity, let's consider a mole of ideal gas that has thermal energy transferred to it. If the volume is held constant, the gas does no work; the thermal energy is absorbed so that all of the thermal energy goes into speeding up the molecules. Since the temperature is directly related to the average speed of the gas molecules, all the added thermal energy goes to raising the temperature of the gas. This is *not* the case for the situation when the pressure of the gas is held constant. Some of the transferred thermal energy is used up in allowing the gas to expand and hence do work on its surroundings. Less energy is left over to speed up the molecules. Hence at constant pressure, the temperature rise is less than it is at constant volume. Thus, C_P is greater than C_V.

18.13. ADIABATIC CHANGES AND THE P-V DIAGRAM

To understand the ideal heat engine proposed by Carnot, we will calculate the work done when a monatomic ideal gas expands or is compressed adiabatically so that no thermal energy is transferred to or from the gas. In general, as a gas expands to a new volume and does work, the pressure is not constant.

$$W^{\text{isothermal}} = nRT\int_{V_1}^{V_2} \frac{1}{V}\,dV = nRT\ln\left(\frac{V_2}{V_1}\right)$$

Hints: (1) Consider why it is legitimate to pull the *n, R,* and *T* terms out of the integral for any isothermal expansion or compression. (2) What is the expression for the integral of *dV/V* that you derived in Activity 18.4.1d?

b. Calculate the work done when one mole of a 300 K gas expands isothermally from an initial pressure of $2.49 \times 10^3\ \mathrm{N/m^2}$ and volume of $1.00\mathrm{m}^3$ to a final pressure of $8.31 \times 10^2\,\mathrm{N/m^2}$ and volume of $3.00\mathrm{m}^3$.

18.15. THE CARNOT ENGINE CYCLE

Let us return to a consideration of the Carnot cycle, which can be shown to be the most efficient possible heat engine cycle. It consists of four elements pictured below on a *P-V* diagram: (1) work done by the gas in an isothermal expansion from $A \mapsto B$ in a piston at T_{hot}; (2) work done by the gas in an adiabatic expansion from $B \mapsto C$ in which the gas is allowed to cool to T_{cold}; (3) work done on the gas in an isothermal compression of the gas from $C \mapsto D$ at T_{cold}; and (4) work done on the gas in an adiabatic compression of the gas from $D \mapsto A$ A while temperature rises back to T_{hot}.

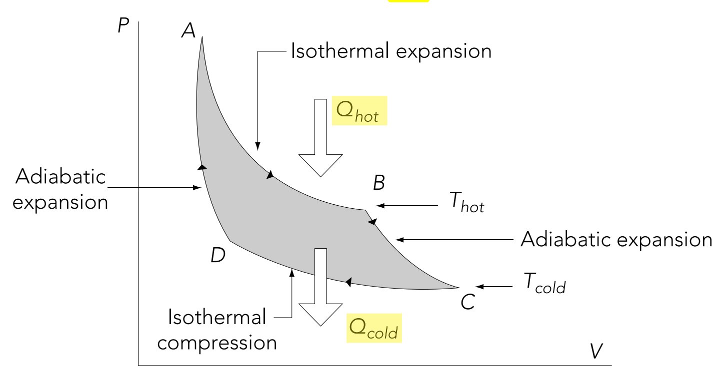

Fig. 18.14. A Carnot cycle consisting of two adiabatic and two isothermal processes.

A Sample Carnot Cycle

Here is a specific example of a Carnot cycle involving 1.00 moles of an ideal monatomic gas for which g = 5/3. It has four "legs." You will be using this sample cycle data in Activity 18.15.1 to make a series of specific calculations that should help you understand the relationship between the thermal energy transfers and the temperatures of the reservoirs for a Carnot engine.

Isothermal Expansion $A \mapsto B$

Point A: The gas is confined to a volume of 1.00 m^3 and a pressure of 2.49×10^3 N/m^2. It is initially at equilibrium with a heat reservoir at a temperature of 300 K. (A heat reservoir is a source of energy that is recharged so it stays at the same temperature no matter how much thermal energy is transferred from it.)

Point B: The gas is allowed to do work on its surroundings by expanding isothermally to a new volume of 3.00 m^3 and a pressure of 8.31×10^2 N/m^2.

Adiabatic Expansion $B \mapsto C$

Point C: The gas is thermally isolated by wrapping the piston in an insulating material and is allowed to do more work and expand further adiabatically until it has cooled to a temperature of 200 K. In this adiabatic process the pressure drops to 3.02×10^2 N/m^2 and the volume increases to 5.51 m^3.

Isothermal Compression $C \mapsto D$

Point D: The gas is placed in thermal contact with a heat reservoir at 200 K and work is done to compress it isothermally to a volume of 1.84 m^3 at an increased pressure of 9.05×10^2 N/m^2.

Adiabatic Compression $D \mapsto A$

Point A: Again: The gas is isolated thermally by insulating it. Then work is done on it to compress it until it reaches a temperature of 300 K and a volume of 1.00 m^3 once again.

18.15.1. Activity: Carnot Cycle Analysis

a. Calculate the ΔE^{int}, Q, and W values for each of the parts of the sample Carnot cycle. Make use of the First Law ($\Delta E^{int} = Q - W$) when you can and recall that $\Delta E^{int} = nC_V\Delta T$. Show the equations and calculations and then summarize the results in the blanks that follow:

1. Isothermal Expansion $A \mapsto B$. **Hints:** Recall that ΔE^{int} can be calculated from the temperature change from A to B. You should be able to use the isothermal work equation and calculations you did in Activity 18.14.2 to determine that $W_{AB} = 2740$ J.

$A \mapsto B$:

ΔE^{int} = __________ J Q = __________ J W = __________ J

2. Adiabatic Expansion $B \mapsto C$. **Hint:** You can use the fact that no thermal energy is transferred to the engine so Q = 0 J and that ΔE^{int} can be calculated from the known temperature change between points B and C.

$B \mapsto C$:

ΔE^{int} = __________ J Q = __________ J W = __________ J

3. Isothermal Compression $C \mapsto D$

$C \mapsto D$:

ΔE^{int} = __________ J Q = __________ J W = __________ J

4. Adiabatic Compression $D \mapsto A$

$D \mapsto A$:

ΔE^{int} = __________ J Q = __________ J W = __________ J

b. Show that the efficiency of this Carnot cycle is η = 0.33. Write the equation that defines heat engine efficiency and also show your calculations.

c. Compare the quantities listed below:

$|Q_{hot}|$ = __________ J T_{hot} = __________ K $\frac{|Q_{hot}|}{T_{hot}}$ = __________ $\frac{J}{K}$

$|Q_{cold}|$ = __________ J T_{cold} = __________ K $\frac{|Q_{cold}|}{T_{cold}}$ = __________ $\frac{J}{K}$

d. Do you see any relationships between the thermal energy transfers and the temperatures tabulated above? Explain.

e. Can you rewrite the efficiency of your Carnot cycle in terms of the temperature of the two reservoirs?

18.16. THE CARNOT EFFICIENCY

From your investigation of the Carnot cycle you should have discovered that

$$\frac{|Q_{hot}|}{|Q_{cold}|} = \frac{T_{hot}}{T_{cold}} \quad \text{so that} \quad \frac{|Q_{hot}|}{|T_{hot}|} = \frac{|Q_{cold}|}{T_{cold}}$$

Carnot recognized that this meant that efficiency, η ("eta"), of his ideal cycle could be described by the equations

$$\eta_{Carnot} = \frac{|W|}{|Q_{hot}|} = \frac{|Q_{hot}| - |Q_{cold}|}{|Q_{hot}|} = 1 - \frac{|Q_{cold}|}{|Q_{hot}|} = 1 - \frac{T_{cold}}{T_{hot}}. \qquad (18.13)$$

Thus, the efficiency of a Carnot engine *depends only on the temperature ratio* between the hot and the cold reservoir. The bigger the ratio, the more efficient the engine. This increase in efficiency with increasing temperature differences holds true for other heat engine cycles, but no cycle has ever been found that is *more* efficient than the Carnot cycle for a given T_{cold}/T_{hot}. What is the secret behind the Carnot cycle's efficiency? In order to answer this question scientists have introduced a new concept called entropy and studied how it changes during various engine cycles. Unfortunately, we do not have time to develop this concept.

18.17. THE STIRLING ENGINE AND THE SECOND LAW OF THERMODYNAMICS

The Stirling Engine

Carnot began working on engines in hopes of improving the efficiency of the steam engine. Although his concept of the ideal heat engine was a rare achievement that laid the groundwork for the first and second laws of thermodynamics, the internal combustion engine used in the cars we drive is far from ideal in its efficiency. The Stirling Engine Cycle proposed by the Reverend Robert Stirling of the Church of Scotland in 1816 is considerably closer in its design to the Carnot engine. In a Stirling engine a piston linked to a displacement system shuffles gas back and forth between hot and cold reservoirs. The expansion and contraction of the gas as it is heated and cooled drives the engine. In the Stirling engine waste heat (thermal energy) transferred back to the engine's surroundings is recycled in an ingenious way that improves efficiency. A modern working model of the early Stirling engine enables us to explore the quantitative behavior of a Carnot-like engine.

For observing the operation of the Stirling engine you will need:

EITHER:

- 1 miniature Stirling engine
- 6 oz. of denatured alcohol
- 2 ice cubes

AND/OR

- 1 Visible Stirling Engine
- 1 ceramic coffee mug
- hot water

Recommended Group Size:	All	Interactive Demo OK?:	Y

18.17.1. Activity: Stirling Engine Efficiency

a. Follow the instructions that come with the Stirling engines and operate it. Examine the engine and try to explain the elements of a basic cycle of the engine. Where is the hot reservoir? The cold reservoir?

b. Assuming that the equation describing the efficiency of the engine is approximately the same as that for the Carnot engine so that

$$\eta_{Carnot} = 1 - \frac{T_{cold}}{T_{hot}}$$

what do you predict will happen to the engine if an ice cube is placed in contact with the cold reservoir?

c. Place the ice cube in contact with the cold reservoir and describe what actually happens to the operation of the engine.