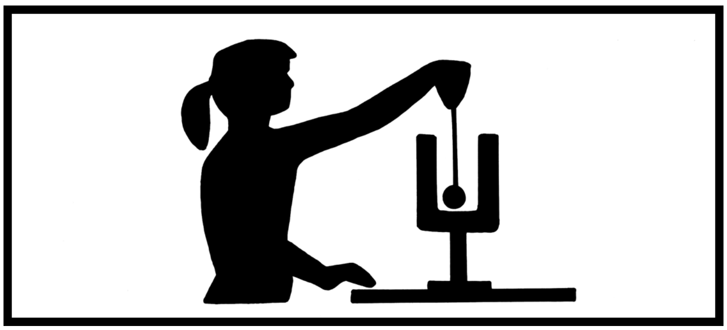
UNIT 20: ELECTRIC FLUX AND GAUSS' LAW



. . . before Maxwell people considered physical reality. . . . as material points. . . .
After Maxwell they considered physical reality as continuous fields. . . .
Albert Einstein

OBJECTIVES

1. To understand how electric field lines and electric flux can be used to describe the magnitude and direction of the electric field in a small region in space.

2. To discover how the electric flux at a small area is related to the magnitude and direction of the area relative to the magnitude and direction of the electric field lines.

3. To discover the relationship between the flux at a "closed surface" and the charge enclosed by that surface for a two-dimensional situation (Gauss' law in Flatland).

4. To review how the expression $\oint \vec{E} \cdot d\vec{A}$ over a closed three-dimensional surface is proportional to the number of field lines passing through the closed surface and thus to extend the discovery of Gauss' law in Flatland to three dimensions.

5. To explore the concept of symmetry.

6. To learn to use Gauss' law to calculate the electric fields that result from highly symmetric distributions of electric charge at various points in space.

By convention, if an electric field line passes from the inside to the outside of a surface, we say the flux is positive. If the field line passes from the outside to the inside of a surface, the flux is negative.

How does the flux at a surface depend on the angle between the normal vector to the surface and the electric field lines? In order to answer this question in a concrete way, you can make a mechanical model of some electric field lines and of a surface. What happens to the electric flux as you rotate the surface at various angles between 0 degrees (or 0 radians) and 180 degrees (or π radians) with respect to the electric field vectors? To make your model you will need to arrange nails in a 10 × 10 array poking up at 1/4″ intervals through a piece of Styrofoam. The surface can be a copper loop painted white on the "outside." You will need:

- Styrofoam or 3/8" plywood (5" × 5" square)
- 100 nails, approximately 4" in length (mounted on the Styrofoam or plywood)
- 1 wire loop (4" × 4" square)
- 1 paper, 5" × 5" with 1/4" graph rulings (to affix to the mounting square to help with spacing the nails)
- 1 protractor

Recommended Group Size:	4	Interactive Demo OK?:	N

Once the model is made you can perform the measurements with a protractor and enter the angle in radians and the flux into a computer data table for graphical analysis.

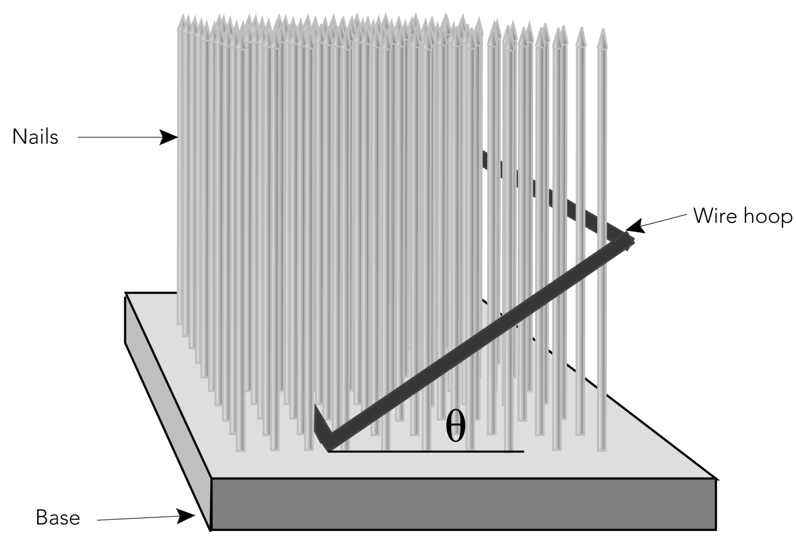


Fig. 20.3. Apparatus designed to determine how many uniformly spaced electric field lines will pass through an imaginary surface area as a function of the angle between the direction of the electric field lines and the normal vector representing the surface area.

c. Try to confirm your guess by constructing a spreadsheet model and overlay graph of the data and the mathematical function you think matches the data. Affix or sketch the plots in the space to the right of the data table. Summarize your procedures and conclusions in the following space.

d. What is the definition of the vector dot product of two vectors in terms of vector magnitudes and the angle θ between them? Can you relate the scalar value of the flux, Φ, to the dot product of the vectors $\vec{E}$ and $\vec{A}$?

20.4. A MATHEMATICAL REPRESENTATION OF FLUX AT A SURFACE

One convenient way to express the relationship between angle and flux for a uniform electric field $\vec{E}$ is to use the dot product so that the flux at a surface $\vec{A}$ is $\Phi = \vec{E} \cdot \vec{A}$. Flux is a scalar. If the electric field is not uniform or if the surface subtends different angles with respect to the electric field lines, then we must calculate the net or total flux by breaking the surface into infinitely many infinitesimal areas, $d\vec{A}$, so that $d\Phi = \vec{E} \cdot d\vec{A}$, and then taking the integral to sum all the flux elements. This gives a net flux of

$$\Phi^{\text{net}} = \int d\Phi = \int \vec{E} \cdot d\vec{A} \quad (\text{net flux at a surface})$$

Some surfaces, like that of a sphere or the series of surfaces that make a rectangular box, are closed surfaces. A *closed surface* has no holes or edges so that nothing can leave its interior without passing through the surface itself. Because we want to study the amount of flux at closed surfaces, there is a special notation to represent the integral of $\vec{E} \cdot d\vec{A}$ at a closed surface. It is represented as follows:

$$\Phi = \oint d\Phi = \oint \vec{E} \cdot d\vec{A} \quad (\text{net flux at a closed surface})$$