The concept of *charge density* is very useful in figuring out how much charge is contained within a given radius r in a charged sphere. Before you get started with your derivations of E-field equations, take a moment to read about the concept of charge density.

If a volume element dV contains a small amount of charge dq, then the charge density is given by the equation:

$$\rho = \frac{dq}{dV}$$

Thus, if the charge is distributed in a spherically symmetric manner, the amount of charge contained within a radius r is given by:

$$q = \iiint \rho\, dV = \int_0^r \rho 4\pi r'^2\, dr' = 4\pi \int_0^r r'^2\, dr'$$

In the case where the charge is uniformly distributed, ρ is a constant given by:

$$\rho = \frac{3Q}{4\pi R^3}$$

where Q is the total charge in the sphere and R is the sphere's radius. In this case, then, the charge contained within a radius r is given by:

$$q = \iiint \rho\, dV = 4\pi\rho \int_0^r r'^2\, dr'$$

Gauss' Law and the Point Charge

Let's begin by using Gauss' law in the form

$$\Phi_e = \oint \vec{E} \bullet d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

to find the electric field magnitude at any distance, r, from a point charge $+q$.

20.9.4. Activity: Gauss' Law and a Point Charge

a. Write down the general expression for Gauss' law.

b. Explain why the dot product $\vec{E} \bullet d\vec{A}$ can be replaced with the expression EdA inside the integral where E is the magnitude of the electric field and dA is the magnitude of an element of area on the surface of the spherical shell. **Hint:** What is the angle between the vector $\vec{E}$ and the vector $d\vec{A}$ at any point on the spherical surface?

c. Explain why the magnitude of the electric field is the same at all points on the spherical shell. **Hint:** Think of Coulomb's law and the definition of electric field.

d. If the magnitude of the electric field, E, is the same at all points on the spherical surface, explain why E can be factored out of the integral $\oint \vec{E} \bullet d\vec{A}$.

e. Since $4\pi r^2$ is the equation for the area of the surface of a sphere, what is the equation for $\oint dA$?

f. Finally, what is the magnitude of the electric field, E, as a function of the central charge, q, and the distance from it, r?

Gauss' Law and the Spherically Symmetric Charge Distribution

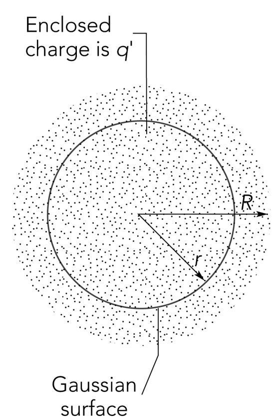
Use some of the ideas from the last two activities about charge density and about using Gauss' law to find the magnitude of the electric field near a point charge to find the values of the magnitude of electric field in the vicinity of a spherically symmetric distribution of charges.

In the next activity you are to compute the magnitude of the electric field at a distance r from the center of a charged sphere of radius R with a total excess charge of q distributed uniformly throughout its volume, where $r < R$ (that is, the point in question is inside the sphere!).

20.9.4. Activity: Gauss' Law and Spherically Symmetric Charge Distributions

a. To use Gauss' Law to find the electric field, you'll need to know the charge enclosed inside a sphere of radius r in which charges are distributed uniformly throughout the larger sphere of radius R. Start by using the equation for the volume of a sphere of radius r ($V = 4/3\ \pi r^3$) to show that the smaller amount of excess charge q' that lies within a radius r is given by

$$q' = q\frac{r^3}{R^3}$$



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