ELECTRICAL AND GRAVITATIONAL FORCES

21.2. COMPARISON OF ELECTRICAL AND GRAVITATIONAL FORCES

Let's start our discussion of this comparison with the familiar expression of the Coulomb force exerted on charge B by charge A.

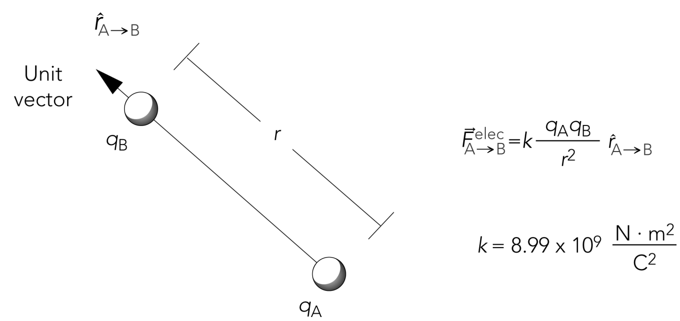

Fig. 21.4. This diagram shows how the directions of unit vector $\hat{r}_{A\to B}$ points in the direction from charge q_A to charge q_B.

Charles Coulomb did his experimental investigations of this force in the eighteenth century by exploring the forces between two small charged spheres. Much later, in the twentieth century, Coulomb's law enabled scientists to design cyclotrons and other types of accelerators for moving charged particles in circular orbits at high speeds.

Newton's discovery of the universal law of gravitation came the other way around. He thought about orbits first. This was back in the seventeenth century, long before Coulomb began his studies. A statement of Newton's universal law of gravitation describing the force experienced by object B due to the presence of object A is shown as follows in modern mathematical notation:

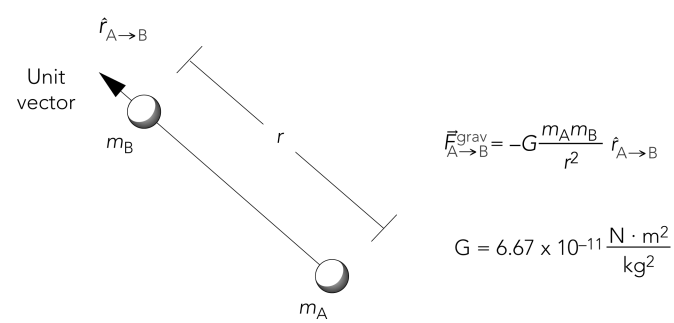

Fig. 21.5. This diagram shows how the direction of unit vector $\hat{r}_{A\to B}$ points from mass A to mass B.

About the time that Coulomb did his experiments with electrical charges in the eighteenth century, one of his contemporaries, Henry Cavendish, did a direct experiment to determine the nature of the gravitational force between two spherical masses in a laboratory. This confirmed Newton's gravitational

Let's peek into the hydrogen atom and compare the gravitational force on the electron due to interaction of its mass with that of the proton to the electrical force between the two particles as a result of their charge. In order to do the calculation you'll need to use some well-known constants.

Electron: $m_e = 9.11 \times 10^{-31}$ kg $q_e = -1.60 \times 10^{-19}$ C

Proton: $m_p = 1.67 \times 10^{-27}$ kg $q_p = +1.60 \times 10^{-19}$ C

Distance between the electron and proton: $r \approx 1.0 \times 10^{-10}$ m

21.3.1. Activity: The Electrical vs. the Gravitational Force in the Hydrogen Atom

a. Calculate the magnitude of the electrical force on the electron. Is it attractive or repulsive?

b. Calculate the magnitude of the gravitational force on the electron. Is it attractive or repulsive?

c. Which is larger? By what factor (that is, what is the ratio)?

d. Which force are you more aware of on a daily basis? If your answer does not agree with that in part c, explain why.

21.4. GAUSS' LAW FOR ELECTRICAL AND GRAVITATIONAL FORCES

Gauss' law states that the net electric flux at any closed surface is equal to the net charge inside the surface divided by the electric constant in a vacuum, ε_0, where $\varepsilon_0 = 1/(4\pi k_e)$. Mathematically this is represented by an integral of the dot product of the electric field and the normal to each element of area over the closed surface:

$$\Phi^{net} = \oint \vec{E} \cdot d\vec{A} = \frac{q^{enc}}{\varepsilon_0}$$

The fact that electric field lines spread out so that their density (and hence the strength of the electric field) decreases at the same rate that the area of an enclosing surface increases can ultimately be derived from the $1/r^2$ dependence of electrical force on distance. Thus, *Gauss' law should also apply to gravitational forces.*

21.4.1. Activity: The Gravitational Gauss' Law

State a gravitational version of Gauss' law in words and then represent it with an equation. **Hint:** The electric field vector, $\vec{E}$, is defined as $\vec{F}^{\text{elec}}_{s\to t'}/q_{t'}$, where $\vec{F}^{\text{elec}}_{s\to t'}$ is the force that source charges exert on a tiny test charge located at a point in space of interest. Let's define the gravitational field vector $\vec{Y}$ as $\vec{F}^{\text{grav}}/m$ by analogy.

We can use the new version of Gauss' law to calculate the gravitational field at some distance from the surface of the earth just as we can use the electrical Gauss' law to determine the electric field at some distance from a uniformly charged sphere. This is useful in figuring out the familiar force "due to gravity" near the surface of the earth and at other locations.

21.4.2. Activity: The Gravitational Force of the Earth

a. Use Gauss' law to show that the magnitude of the gravitational field, $\vec{Y}$, at a height h above the surface of the earth is given by $GM/(R+h)^2$, where M is the Earth's mass and R is the radius. **Hints:** How much mass is enclosed by a spherical shell of radius $R+h$? Does $\vec{Y}$ have a constant magnitude everywhere on the surface of the spherical shell? Why? If so, can you pull it out of the Gauss' law integral?

b. Calculate the gravitational field, $\vec{Y}$, at the surface of the earth where $h = 0.00$ m. Assume the radius of the earth is $R \approx 6.38 \times 10^3$ km and its mass $M \approx 5.98 \times 10^{24}$ kg. Does the result look familiar? How is $\vec{Y}$ related to the local gravitational constant given by $g = 9.8\,\text{N/kg}$?