Vector Notation

Overview

An arrow over a variable indicates it is a vector.

$$\vec{F}^{net} = m\vec{a} \tag{1}$$

The magnitude of the vector is indicated by either F^{net} (no arrow drawn) or $|\vec{F}^{net}|$ (absolute value brackets written around the vector).

A vector can be decomposed into component vectors:

$$\begin{aligned} \vec{a} &= a_x\ \hat{i} + a_y\ \hat{j} + a_z\ \hat{k} \qquad (2) \\ \vec{a} &= a_x\ \hat{x} + a_y\ \hat{y} + a_z\ \hat{z}, \qquad (3) \end{aligned}$$

where $a_x\ \hat{i}$, $a_y\ \hat{j}$, and $a_z\ \hat{k}$ are the component vectors. The value of the component (e.g. a_x) can be positive or negative. Some books use $\hat{x}, \hat{y}, \hat{z}$ to indicate the unit vectors. Others use $\hat{i}, \hat{j}, \hat{k}$. Both are valid.

Some important notes:

- The value of each component vector (e.g. a_x) can be negative or positive. The magnitude of a_x would be written as $|a_x|$.
- The gravitational constant g, is a positive scalar quantity, $g = 9.8$ N/kg.
- Different books use different variables to indicate the same forces. These are summarized on Page 214 of your Activity Guide, Section 7.14. I don't care which you use (such as $\vec{T}$ vs. $\vec{F}^{tens}$), as long as you are consistent within an assigned problem.

- If the value of a vector component is zero, it is not necessary to write that out. For example, for the force of gravity in the negative y direction, it is acceptable to write $\vec{F}^{grav} = -mg\ \hat{j}$. You don't need to write $\vec{F}^{grav} = 0\ \hat{i} - mg\ \hat{j}$ (though it would not be incorrect).

Examples

Here are many different ways to express the force acting upon a ball in free fall. You'll never need to write all of these at once; they are just meant to show the variety of ways to use correct notation. Here the positive direction of the y-axis points up. m and g are both positive scalar quantities.

$$\begin{aligned}
\vec{F}^{grav} &= F_y^{grav}\hat{j} && (4)\\
\vec{F}^{grav} &= -mg\hat{j} && (5)\\
F_y^{grav} &= -mg && (6)\\
|F_y^{grav}| &= mg \text{ (Absolute value)} && (7)\\
F^{grav} &= mg \text{ (Absolute value)} && (8)\\
|\vec{F}^{grav}| &= mg \text{ (Absolute value)} && (9)
\end{aligned}$$

Let's say the ball has reached terminal velocity. This is when the force of air resistance is equal and opposite to the force of gravity, so the ball no longer accelerates, because the net force is zero. There are a few ways to write that.

$$\begin{aligned}
\vec{F}^{net} &= \vec{F}^{grav} + \vec{F}^{air} = 0 && (10)\\
\vec{F}^{grav} &= -\vec{F}^{air} && (11)\\
F_y^{grav} &= -F_y^{air} \text{ (Scalar vector components)} && (12)\\
F^{grav} &= F^{air} \text{ (Absolute values)} && (13)\\
|F_y^{grav}| &= |F_y^{air}| \text{ (Absolute values)} && (14)
\end{aligned}$$

Breaking a Vector Into its Components

Imagine a force vector, $\vec{F}$, that is at an angle θ relative to our x axis.

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} \tag{15}$$

$$\vec{F} = F\cos(\theta)\hat{\imath} + F\sin(\theta)\hat{\jmath} \tag{16}$$

$$F^2 = F_x^2 + F_y^2 \text{ (Magnitudes figured via Pythagorean Theorem)} \tag{17}$$

When the angle relative to the x axis is greater than $90°$, you have a couple of options. One option is to figure out the angle relative to the x-axis and continue to use the general $\hat{r} = \cos(\theta)\hat{\imath} + \sin(\theta)\hat{\jmath}$ formula. The positive and negative signs will work themselves out if you have chosen your angle correctly.

The other option is to draw your own triangle to determine the magnitudes of the vector components, using an angle which might be different than the angle with respect to the x-axis. Then manually determine which vector components are positive or negative.